Quiz 2

Name: Solution Key

Let

$$S = \left\{ \mathbf{v}_1 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}, \mathbf{v}_4 = \begin{bmatrix} 11 \\ 1 \\ 6 \end{bmatrix}, \mathbf{v}_5 = \begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix} \right\} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} -9 \\ 6 \\ 1 \end{bmatrix}.$$

1. Determine if vector **b** belongs to the span of vectors in set S? If yes, then express **b** as a linear combination of vectors in set S.

Let $A = [v_1, v_2, v_3, v_4, v_5]$. Check if $Ax = b$ is consistent.

$$[A|b] = \begin{bmatrix} 2 & 4 & 1 & 11 & -3 & -9 \\ 1 & 2 & -1 & 1 & 1 & 6 \\ 3 & 6 & -2 & 6 & -2 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & -1 & 1 & 1 & 6 \\ 2 & 4 & 1 & 11 & -3 & -9 \\ 3 & 6 & -2 & 6 & -2 & 1 \end{bmatrix} \begin{matrix} R_2 \\ R_1 \\ R_3 \end{matrix}$$

$$\to \begin{bmatrix} 1 & 2 & -1 & 1 & 1 & 6 \\ 0 & 0 & 3 & 9 & -5 & -21 \\ 0 & 0 & 1 & 3 & -5 & -17 \end{bmatrix} \begin{matrix} \\ R_2 - 2R_1 \\ R_3 - 3R_1 \end{matrix} \to \begin{bmatrix} 1 & 2 & -1 & 1 & 1 & 6 \\ 0 & 0 & 1 & 3 & -5 & -17 \\ 0 & 0 & 3 & 9 & -5 & -21 \end{bmatrix} \begin{matrix} R_1 \\ R_3 \\ R_2 \end{matrix}$$

$$\to \begin{bmatrix} 1 & 2 & -1 & 1 & 1 & 6 \\ 0 & 0 & 1 & 3 & -5 & -17 \\ 0 & 0 & 0 & 0 & 10 & 30 \end{bmatrix} \begin{matrix} \\ \\ R_3 - 3R_2 \end{matrix} \to \begin{bmatrix} 1 & 2 & -1 & 1 & 1 & 6 \\ 0 & 0 & 1 & 3 & -5 & -17 \\ 0 & 0 & 0 & 0 & 1 & 3 \end{bmatrix}$$

Yes, $b \in \text{span}(S)$.

$$\to \begin{bmatrix} 1 & 2 & -1 & 1 & 0 & 3 \\ 0 & 0 & 1 & 3 & 0 & -2 \\ 0 & 0 & 0 & 0 & 1 & 3 \end{bmatrix} \begin{matrix} R_1 - R_3 \\ R_2 + 5R_3 \\ \end{matrix} \to \begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 & \end{matrix} \\ \left[\begin{array}{ccccc|c} 1 & 2 & 0 & 4 & 0 & 1 \\ 0 & 0 & 1 & 3 & 0 & -2 \\ 0 & 0 & 0 & 0 & 1 & 3 \end{array}\right] \end{array} \begin{matrix} R_1 + R_2 \\ \text{(RREF)} \\ \end{matrix}$$

$$\begin{cases} x_5 = 3 \\ x_4 = 0 \text{ (Free, set to zero)} \\ x_3 = -2 - 3x_4 = -2 \\ x_2 = 0 \text{ (Free, set to zero)} \\ x_1 = 1 - 2x_2 - 4x_4 = 1. \end{cases} \qquad \bar{x} = \begin{pmatrix} 1 \\ 0 \\ -2 \\ 0 \\ 3 \end{pmatrix}.$$

$$\bar{b} = v_1 - 2v_3 + 3v_5.$$

2. Give a linearly independent subset of S that generates span(S).

$\{v_1, v_3, v_5\}$ (Pivot columns of A).

3. Does the set S generate R^3? Explain.

Yes, matrix A has a pivot in every row.