

September 2, 2009

## Quiz 1

Name: Solution Key

1. Determine whether a linear system is consistent, and if it is consistent, then find all of its solutions.

$$x_1 + 2x_2 + x_3 - 3x_4 + 3x_6 = 0$$

$$-2x_1 - 4x_2 + x_3 - 3x_4 = 3$$

$$-x_1 - 2x_2 + 2x_3 - 6x_4 + x_5 + 6x_6 = 4$$

$$[A|b] = \begin{bmatrix} 1 & 2 & 1 & -3 & 0 & 3 & 0 \\ -2 & -4 & 1 & -3 & 0 & 0 & 3 \\ -1 & -2 & 2 & -6 & 1 & 6 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 1 & -3 & 0 & 3 & 0 \\ 0 & 0 & 3 & -9 & 0 & 6 & 3 \\ 0 & 0 & 3 & -9 & 1 & 9 & 4 \end{bmatrix} \begin{array}{l} R_2 + 2R_1 \\ R_3 + R_1 \end{array}$$

$$\rightarrow \begin{bmatrix} 1 & 2 & 1 & -3 & 0 & 3 & 0 \\ 0 & 0 & 1 & -3 & 0 & 2 & 1 \\ 0 & 0 & 3 & -9 & 1 & 9 & 4 \end{bmatrix} \xrightarrow{\frac{1}{3}R_2} \begin{bmatrix} 1 & 2 & 1 & -3 & 0 & 3 & 0 \\ 0 & 0 & 1 & -3 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 1 & 3 & 1 \end{bmatrix} \leftarrow \text{REF}$$

No pivot in the last column  $\Rightarrow$  system is consistent.

$$\rightarrow \begin{bmatrix} \textcircled{1} & 2 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & \textcircled{1} & -3 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & \textcircled{1} & 3 & 1 \end{bmatrix} \begin{array}{l} R_1 - R_2 \\ \leftarrow \text{RREF} \end{array}$$

 $x_1, x_3, x_5$  - basic variables,  $x_2, x_4, x_6$  free variables.

$$\begin{cases} x_6 = t \\ x_5 = 1 - 3t \\ x_4 = u \\ x_3 = 1 + 3u - 2t \\ x_2 = v \\ x_1 = -1 - 2v - t \end{cases} \Rightarrow \bar{x} = \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} -1 \\ 0 \\ -2 \\ 0 \\ -3 \\ 1 \end{pmatrix} + u \begin{pmatrix} 0 \\ 0 \\ 3 \\ 1 \\ 0 \\ 0 \end{pmatrix} + v \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, u, t, v \in \mathbb{R}$$

2. List pivot positions of the augmented matrix of the linear system.

$$(1,1), (2,3), (3,5).$$

3. List indices of pivot columns of the augmented matrix.

$$1, 3, 5. - \text{pivot columns.}$$