July 24, 2008 **Quiz 4** Name: Solution Key

1. Let $A = \begin{pmatrix} 1 & 2 & -1 & 0 & 3 \\ 1 & 2 & 0 & 2 & -1 \\ 2 & 4 & 3 & 2 & 7 \end{pmatrix}$.

(a) Find a basis for the null space of matrix A.

$$\begin{pmatrix} 1 & 2 & -1 & 0 & 3 \\ 1 & 2 & 0 & 2 & -1 \\ 2 & 4 & 3 & 2 & 7 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & -1 & 0 & 3 \\ 0 & 0 & 1 & 2 & -4 \\ 0 & 0 & 5 & 2 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & -1 & 0 & 3 \\ 0 & 0 & 1 & 2 & -4 \\ 0 & 0 & 0 & -8 & 21 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & -1 & 0 & 3 \\ 0 & 0 & 1 & 2 & -4 \\ 0 & 0 & 0 & 1 & -\frac{21}{8} \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 2 & -1 & 0 & 3 \\ 0 & 0 & 1 & 0 & 5/4 \\ 0 & 0 & 0 & 1 & -21/8 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 0 & 0 & 17/4 \\ 0 & 0 & 1 & 0 & 5/4 \\ 0 & 0 & 0 & 1 & -21/8 \end{pmatrix}$$

$\frac{21}{4} - 4 = \frac{21-16}{4} = \frac{5}{4}$, $3 + \frac{5}{4} = \frac{12+5}{4} = \frac{17}{4}$.

x_1, x_3, x_4 - basic, x_2, x_5 - free

$x_5 = s$
$x_4 = \frac{21}{8} x_5 = \frac{21}{8} s$
$x_3 = -\frac{5}{4} x_5 = -\frac{5}{4} s$
$x_2 = t$
$x_1 = -17/4\, s - 2t$

$$\bar{x} = s \begin{pmatrix} -17/4 \\ 0 \\ -5/4 \\ 21/8 \\ 1 \end{pmatrix} + t \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Basis for $N(A)$

(b) Find a basis for the column space of matrix A.

Pivot Columns of A is a basis for $Col(A)$ $\Rightarrow \left\{ \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix} \right\}$.

(c) Find a basis for the row space of matrix A.

Pivot Rows of REF is a basis for $Row(A)$ $\Rightarrow \{(1, 2, 0, 0, 17/4), (0, 0, 1, 0, 5/4), (0, 0, 0, 1, -21/8)\}$.

(d) Specify the nullity of matrix A?

Nullity (A) = # free variables = 2

(e) Specify rank of matrix A?

rank (A) = # basic variables = 3

2. The set $\beta = \{1+t,\ 1-3t, 1+t+t^2\}$ is a basis for the vector space P_2. Find the β-coordinate vector of the polynomial $p(t) = 3 + 11t + 3t^2$ and verify your result. Let γ be the standard basis, $\gamma = \{1, t, t^2\}$.

$P_{\gamma \leftarrow \beta} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -3 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, $[p]_\gamma = \begin{bmatrix} 3 \\ 11 \\ 3 \end{bmatrix}$. $P_{\gamma \leftarrow \beta} [p]_\beta = [p]_\gamma \Rightarrow$

$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -3 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} 3 \\ 11 \\ 3 \end{bmatrix}$ $\Leftarrow$ Solve.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & -3 & 1 & 11 \\ 0 & 0 & 1 & 3 \end{array}\right] \to \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -4 & 0 & 8 \\ 0 & 0 & 1 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 1 & 5 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$x_1 = 2$, $x_2 = -2$, $x_3 = 3 \Rightarrow [p]_\beta = \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}$.

<u>Verify:</u> $2 \cdot (1+t) - 2(1-3t) + 3(1+t+t^2) = (2-2+3) + t(2+6+3) + t^2 \cdot 3$
$= 3 + 11t + 3t^2 = p(t)$.