Quiz 3 **Name:** Solution Key

1. Compute the determinant using row-reduction:

$$\begin{vmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ -2 & 1 & -2 & 6 \\ 0 & 3 & 4 & -5 \end{vmatrix} \underset{\text{transpose}}{\overset{\uparrow}{=}} \begin{vmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & -2 & 4 \\ 1 & 2 & 6 & -5 \end{vmatrix} = \begin{vmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & -2 & 4 \\ 0 & 2 & 8 & -5 \end{vmatrix} = \begin{vmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & -2 & 4 \\ 0 & 0 & 6 & -11 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & -2 & 4 \\ 0 & 0 & 0 & 1 \end{vmatrix} = 1\cdot 1\cdot(-2)\cdot 1 = -2$$

2. Compute the determinant in part 1 using co-factor expansion.

$$\begin{vmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ -2 & 1 & -2 & 6 \\ 0 & 3 & 4 & -5 \end{vmatrix} \overset{\text{1st Row}}{=} \begin{vmatrix} 1 & 0 & 2 \\ 1 & -2 & 6 \\ 3 & 4 & -5 \end{vmatrix} + 0 + 0 - \begin{vmatrix} 0 & 1 & 0 \\ -2 & 1 & -2 \\ 0 & 3 & 4 \end{vmatrix}$$

$$= \underbrace{\begin{vmatrix} -2 & 6 \\ 4 & -5 \end{vmatrix} + 2\begin{vmatrix} 1 & -2 \\ 3 & 4 \end{vmatrix}}_{\text{1st row}} \underbrace{-(-1)\begin{vmatrix} -2 & -2 \\ 0 & 4 \end{vmatrix}}_{\text{1st Row}} = 10 - 24 + 2(4+6) + (-8)$$

$$= -2$$

3. Let W be a set of quadratic polynomials such that, for any $p(t) = at^2 + bt + c \in W$, $p(0) = 0$ and $p(1) = 0$. Prove that W is a subspace of P_2.

(1). P_2 is a vector space and W is a subset of P_2.

(2) $\theta(t) \equiv 0 \in W$? $\theta(0) = 0$ and $\theta(1) = 0 \Rightarrow \theta(t) \in W$. Yes.

(3) Let $p(t), q(t)$ be in W: $p(0) = p(1) = q(0) = q(1) = 0$.

~~Let c~~ $(p+q)(t) = p(t) + q(t)$

$$\left.\begin{array}{l} (p+q)(0) = p(0) + q(0) = 0 + 0 = 0 \\ (p+q)(1) = p(1) + q(1) = 0 + 0 = 0 \end{array}\right\} \Rightarrow (p+q)(t) \in W.$$

Note: $W = \text{span}\{t^2 - t\}$.

(4) Let $p(t) \in W$ and $c \in \mathbb{R}$.

$$(cp)(t) = cp(t),\ \forall t, \Rightarrow \left.\begin{array}{l} (cp)(0) = cp(0) = c\cdot 0 = 0. \\ (cp)(1) = cp(1) = c\cdot 0 = 0 \end{array}\right\} \Rightarrow cp \in W.$$

(1), (2), (3), (4) imply W is a subspace of P_2