Quiz 2

Name: Solution Key

Let

$$S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -5 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -7 \end{bmatrix} \right\} \text{ and } \mathbf{b} = \begin{bmatrix} 6 \\ 2 \\ 5 \end{bmatrix}.$$

1. Is the set S linearly dependent or independent? Explain.

Since S consists of 4 vectors and S is a subset of R^3, the set S is linearly dependent (# of vectors is greater than the dimension of the space).

2. Find a lin. ind. subset of S that generates span(S). Row reduce the matrix

$$\begin{bmatrix} 1 & 1 & 4 & 0 \\ 0 & 2 & 2 & 2 \\ 2 & -5 & 1 & -7 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 4 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & -7 & -7 & -7 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 4 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Pivot columns: 1st and 2nd.
Vectors $\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}$ span span(S) and form a linearly independent set.

3. Determine if vector $\mathbf{b}$ belongs to the span of vectors in S? If yes, then express $\mathbf{b}$ as a linear combination of vectors in S.

Consider a linear system $x_1 \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + x_2 \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \\ 5 \end{pmatrix}$.

$$\left[\begin{array}{cc|c} 1 & 1 & 6 \\ 0 & 2 & 2 \\ 2 & -5 & 5 \end{array}\right] \to \begin{bmatrix} 1 & 1 & 6 \\ 0 & 1 & 1 \\ 0 & -7 & -7 \end{bmatrix} \to \left[\begin{array}{cc|c} 1 & 1 & 6 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right] \leftarrow$$

No rows as $[0, 0, *]$, $* \neq 0$ $\Rightarrow$ consistent and $\vec{b} \in$ span(S).

$$\to \left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right] \leftarrow \text{RREF}.$$

$$\Rightarrow \begin{cases} x_1 = 5 \\ x_2 = 1 \end{cases} \Rightarrow \vec{b} = \begin{pmatrix} 6 \\ 2 \\ 5 \end{pmatrix} = 5 \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}.$$

4. Does the set S span R^3? Explain.

No, S doesn't span R^3. REF of the matrix doesn't have a pivot in 3rd row. To be a spanning set for R^3, all rows should have a pivot.