

June 20, 2008

Quiz 1

Name: Solution Key

Determine whether a linear system is consistent, and if it is consistent then find all its solutions.

$$1. \quad \begin{aligned} 2x_1 + 4x_2 + 2x_3 &= 14 \\ 3x_1 - x_3 &= -1 \\ 5x_1 + 5x_2 + 2x_3 &= 17 \end{aligned}$$

$$\begin{bmatrix} 2 & 4 & 2 & 14 \\ 3 & 0 & -1 & -1 \\ 5 & 5 & 2 & 17 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 1 & 7 \\ 3 & 0 & -1 & -1 \\ 5 & 5 & 2 & 17 \end{bmatrix} \xrightarrow{\frac{1}{2}R_1} \begin{bmatrix} 1 & 2 & 1 & 7 \\ 0 & -6 & -4 & -22 \\ 0 & -5 & -3 & -18 \end{bmatrix} \begin{array}{l} R_2 - 3R_1 \\ R_2 - 5R_1 \end{array}$$

$$\rightarrow \begin{bmatrix} 1 & 2 & 1 & 7 \\ 0 & 1 & \frac{2}{3} & \frac{11}{3} \\ 0 & 5 & 3 & 18 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 1 & 7 \\ 0 & 1 & \frac{2}{3} & \frac{11}{3} \\ 0 & 0 & -\frac{1}{3} & -\frac{1}{3} \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 2 & 1 & 7 \\ 0 & 1 & \frac{2}{3} & \frac{11}{3} \\ 0 & 0 & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 0 & 6 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad \text{Linear system is consistent.}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{bmatrix} \Rightarrow \boxed{\begin{array}{l} x_1 = 0 \\ x_2 = 3 \\ x_3 = 1 \end{array}} \leftarrow \text{unique solution.}$$

$$2. \quad \begin{aligned} x_1 - x_2 + 3x_3 &= 2 \\ 2x_1 + x_2 + 2x_3 &= 3 \end{aligned}$$

$$\begin{bmatrix} 1 & -1 & 3 & 2 \\ 2 & 1 & 2 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 3 & 2 \\ 0 & 3 & -4 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 3 & 2 \\ 0 & 1 & -\frac{4}{3} & -\frac{1}{3} \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & \frac{5}{3} & \frac{5}{3} \\ 0 & 1 & -\frac{4}{3} & -\frac{1}{3} \end{bmatrix} \quad \begin{array}{l} \text{Linear system is consistent.} \\ x_1, x_2 \text{ - basic variables} \\ x_3 \text{ is a free variable.} \end{array}$$

Let $x_3 = t$

$$x_2 = -\frac{1}{3} + \frac{4}{3}t$$

$$x_1 = \frac{5}{3} - \frac{5}{3}t$$

$$\Rightarrow \begin{cases} x_1 = \frac{5}{3} - \frac{5}{3}t \\ x_2 = -\frac{1}{3} + \frac{4}{3}t \\ x_3 = t \end{cases} \quad \infty\text{-many solutions.}$$