November 4, 2008 **Quiz 5** **Name:** Solution Key

1. Let $A = \begin{pmatrix} 1 & 2 & 1 & 3 & 5 \\ 2 & 5 & 3 & 1 & 6 \\ 1 & -1 & -2 & 2 & 1 \end{pmatrix}$.

(a) Find a basis for the null space of matrix A. Solve $Ax=0$.

$$A = \begin{pmatrix} 1 & 2 & 1 & 3 & 5 \\ 2 & 5 & 3 & 1 & 6 \\ 1 & -1 & -2 & 2 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 1 & 3 & 5 \\ 0 & 1 & 1 & -5 & -4 \\ 0 & -3 & -3 & -1 & -4 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 1 & 3 & 5 \\ 0 & 1 & 1 & -5 & -4 \\ 0 & 0 & 0 & -16 & -16 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 1 & 3 & 5 \\ 0 & 1 & 1 & -5 & -4 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 2 & 1 & 3 & 5 \\ 0 & 1 & 1 & -5 & -4 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 1 & 0 & 2 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

x_1, x_2, x_4 - basic var.
x_3, x_5 - free

$x_5 = s$
$x_4 = -x_5 = -s$
$x_3 = t$
$x_2 = -x_5 - x_3 = -s - t$
$x_1 = x_3 = t$

$$\bar{x} = s\begin{pmatrix} 0 \\ -1 \\ 0 \\ -1 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ 1 \\ 0 \\ 0 \end{pmatrix}.$$

Basis for $N(A)$ is the set $\left\{ \begin{pmatrix} 0 \\ -1 \\ 0 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \right\}$

(b) Find a basis for the column space of matrix A.

Pivot columns of A form a basis for $Col(A)$ $\Rightarrow \left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \right\}$ - basis for $Col(A)$.

(c) What is the nullity of matrix A?

nullity$(A) = \#$ nonpivot columns $= 2$

(d) What is the rank of matrix A?

rank$(A) = \#$ pivot columns $= 3$

2. Show that the set $\beta = \{t+1,\ t-1, (t-1)^2\}$ is a basis for the vector space P_2.

$\dim(P_2) = 3$. Set β is a basis if vectors in β are linearly independent.
Let $\gamma = \{1, t, t^2\}$ be the standard basis for P_2.
Let $P = [[t+1]_\gamma, [t-1]_\gamma, [t^2-2t+1]_\gamma] = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow$ invertible.
$\Rightarrow \beta$ is a basis for P_2

3. Let β be a basis for P_2 from the previous problem. Find the β-coordinate vector of the polynomial $p(t) = 2t^2 - 5t + 6$ and verify your result.

$[p]_\gamma = \begin{pmatrix} 6 \\ -5 \\ 2 \end{pmatrix}$. Solve $Px = \begin{pmatrix} 6 \\ -5 \\ 2 \end{pmatrix}$, where $P = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$.

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 6 \\ 1 & 1 & -2 & -5 \\ 0 & 0 & 1 & 2 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & -1 & 1 & 6 \\ 0 & 2 & -3 & -11 \\ 0 & 0 & 1 & 2 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & -1 & 0 & 4 \\ 0 & 1 & -3/2 & -11/2 \\ 0 & 0 & 1 & 2 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & -1 & 0 & 4 \\ 0 & 1 & 0 & -5/2 \\ 0 & 0 & 1 & 2 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3/2 \\ 0 & 1 & 0 & -5/2 \\ 0 & 0 & 1 & 2 \end{array}\right)$$

$\Rightarrow [p]_\beta = \begin{pmatrix} 3/2 \\ -5/2 \\ 2 \end{pmatrix}$. Verify: $\frac{3}{2}(t+1) - \frac{5}{2}(t-1) + 2(t^2 - 2t + 1) =$
$= 2t^2 + (\frac{3}{2} - \frac{5}{2} - 4)t + (\frac{3}{2} + \frac{5}{2} + 2) = 2t^2 - 5t + 6 = p(t)$.