Quiz 4 Name: Solution Key

1. Find the volume of the parallelepiped determined by the vectors: $\vec{v}_1 = (1,3,0)$, $\vec{v}_2 = (2,2,-1)$, $\vec{v}_3 = (3,-4,0)$.

$$\pm \text{volume} = \begin{vmatrix} 1 & 3 & 0 \\ 2 & 2 & -1 \\ 3 & -4 & 0 \end{vmatrix} = -1(-1)^{2+3} \begin{vmatrix} 1 & 3 \\ 3 & -4 \end{vmatrix} = -4-9 = -13$$

Volume is 13.

2. Solve the linear system using Cramer's rule: $\begin{cases} 4x - 3y = -3 \\ 6x - 2y = 5 \end{cases}$

Let $A = \begin{pmatrix} 4 & -3 \\ 6 & -2 \end{pmatrix}$ and $b = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$.

$|A| = -8+18 = 10.$

$|A_1| = \begin{vmatrix} -3 & -3 \\ 5 & -2 \end{vmatrix} = 6+15 = 21 \Rightarrow$

$|A_2| = \begin{vmatrix} 4 & -3 \\ 6 & 5 \end{vmatrix} = 20+18 = 38$

$$x = \frac{|A_1|}{|A|} = \frac{21}{10} = 2.1$$

$$y = \frac{|A_2|}{|A|} = \frac{38}{10} = 3.8$$

3. Let P_2 be the vector space of quadratic polynomials, and let W be the subset of P_2 such that

$$W = \{p(t) \in P_2 : \ p(t) = at^2 + bt + (a-b), \forall a, b \in R\}.$$

Is W a subspace of P_2? Explain.

W is a subspace of P_2 if: 1) $\Theta(t) \in W$, 2) Closed under vector add. 3) Closed under scalar multiplication.

Verify 1): Take $a = b = 0 \Rightarrow p(t) = \Theta(t) \in W$.

2) $p(t) = a_1t^2 + b_1t + (a_1 - b_1)$, $q(t) = a_2t^2 + b_2(t) + (a_2 - b_2)$.

$(p+q)(t) = (a_1+a_2)t^2 + (b_1+b_2)t^2 + ([a_1+a_2] - [b_1+b_2]) \in W.$

3) $\alpha p(t) = (\alpha a_1)t + (\alpha b_1)t + (\alpha a_1 - \alpha b_1) \in W.$

W is a subspace.