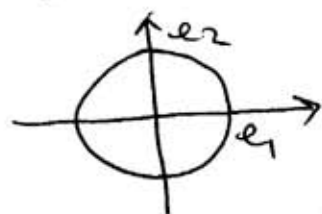


Quiz 3

Name: Solution Key

1. Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the linear transformation that, first, reflects through the origin, and then expands in the x_1 (horizontal) direction by the factor of 3.

a) Find the standard matrix of transformation T ;



$$e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} -1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} -3 \\ 0 \end{pmatrix} = T(e_1)$$

$$e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ -1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ -1 \end{pmatrix} = T(e_2)$$

$$[T] = [T(e_1), T(e_2)] = \begin{bmatrix} -3 & 0 \\ 0 & -1 \end{bmatrix}$$

b) Find the image of the vector $v = (1, -1)$.

$$T \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{bmatrix} -3 & 0 \\ 0 & -1 \end{bmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}.$$

2. A linear transformation T has the following standard matrix $[T] = \begin{pmatrix} 1 & -4 & 3 \\ -2 & 7 & -5 \\ 1 & -5 & 4 \\ 3 & -10 & 5 \end{pmatrix}$.

a) Specify the domain and the co-domain of T .

Since $[T]$ is 4×3 matrix, the domain of T is \mathbb{R}^3 , and the co-domain of T is \mathbb{R}^4 .

b) Determine if the transformation T is onto, one-to-one, or a bijection. Explain.

$$\begin{bmatrix} 1 & -4 & 3 \\ -2 & 7 & -5 \\ 1 & -5 & 4 \\ 3 & -10 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -4 & 3 \\ 0 & -1 & 1 \\ 0 & -1 & 1 \\ 0 & 2 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -4 & 3 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -4 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

T is NOT ONTO - $[T]$ can't have a pivot in every row. Equation $T(x) = b$ is inconsistent for some $b \in \mathbb{R}^4$.

T is one-to-one - every column of T is a pivot column.

The equation $T(x) = 0$ has only the zero solution.

T is NOT a bijection - T is not onto.