Quiz 2

Name: Solution Key

Let

$$S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 8 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 16 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 10 \end{bmatrix} \right\} \text{ and } \mathbf{b} = \begin{bmatrix} 0 \\ -3 \\ 12 \end{bmatrix}.$$

1. Is the set S linearly dependent or independent? Explain.

The set S is linearly dependent because S consists of 5 vectors and S is a subset of R^3 (3-D vectors). $5>3$.

2. Find a linearly independent subset of S that generates span(S).

$$A = \begin{pmatrix} 1 & -1 & 0 & 2 & 1 \\ 2 & -3 & -2 & 1 & 0 \\ 2 & 2 & 8 & 16 & 10 \end{pmatrix} \to \begin{pmatrix} 1 & -1 & 0 & 2 & 1 \\ 0 & -1 & -2 & -3 & -2 \\ 0 & 4 & 8 & 12 & 8 \end{pmatrix} \to \begin{pmatrix} 1 & -1 & 0 & 2 & 1 \\ 0 & 1 & 2 & 3 & 2 \\ 0 & 1 & 2 & 3 & 2 \end{pmatrix} \to \begin{pmatrix} 1 & -1 & 0 & 2 & 1 \\ 0 & 1 & 2 & 3 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The set $S' = \left\{ \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ -3 \\ 2 \end{pmatrix} \right\}$ is linearly independent subset of S that generates span(S).

3. Determine if vector $\mathbf{b}$ belongs to the span of vectors in S? If yes, then express $\mathbf{b}$ as a linear combination of vectors in S.

Let $\bar{v}_1 = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$, $\bar{v}_2 = \begin{pmatrix} -1 \\ -3 \\ 2 \end{pmatrix}$. Is $c_1\bar{v}_1 + c_2\bar{v}_2 = \bar{b}$ consistent?

$$[\bar{v}_1, \bar{v}_2, \bar{b}] = \begin{bmatrix} 1 & -1 & 0 \\ 2 & -3 & -3 \\ 2 & 2 & 12 \end{bmatrix} \to \begin{bmatrix} 1 & -1 & 0 \\ 0 & -1 & -3 \\ 0 & 4 & 12 \end{bmatrix} \to \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right] \to \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

system is consistent since there are no rows as $(0, 0, \beta_{\neq 0})$.

$c_1 = 3$, $c_2 = 3$

$\bar{b}$ belongs to the span(S).

$\bar{b} = 3\bar{v}_1 + 3\bar{v}_2$.

4. Does the set S span R^3? Explain.

The set S <u>does not</u> span R^3 since not every row of matrix A has a pivot entry.