

September 4, 2008

Quiz 1

Name: Solution Key

Determine whether a linear system is consistent, and if it is consistent, then find all its solutions.

$$\begin{aligned}
 1. \quad & x_1 + 2x_2 - 3x_3 = -1 \\
 & 2x_1 + 4x_2 - 6x_3 = -5 \\
 & 4x_1 + x_2 - x_3 = 3
 \end{aligned}$$

$$[A|b] = \begin{bmatrix} 1 & 2 & -3 & -1 \\ 2 & 4 & -6 & -5 \\ 4 & 1 & -1 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & -3 & -1 \\ 0 & 0 & 0 & 1 \\ 4 & 1 & -1 & 3 \end{bmatrix}.$$

Linear system is inconsistent since REF of $[A|b]$ has the row $[0, 0, 0, 1]$ which corresponds to the equation $0=1$, which is false.

$$\begin{aligned}
 2. \quad & x_1 - 2x_2 + 4x_3 - x_4 = 2 \\
 & 3x_1 - 7x_2 + 12x_3 - 4x_4 = 3 \\
 & x_1 + x_2 + x_3 + 2x_4 = 2
 \end{aligned}$$

$$[A|b] = \begin{bmatrix} 1 & -2 & 4 & -1 & 2 \\ 3 & -7 & 12 & -4 & 3 \\ 1 & 1 & 1 & 2 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & 4 & -1 & 2 \\ 0 & -1 & 0 & -1 & -3 \\ 0 & 3 & -3 & 3 & 0 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & -2 & 4 & -1 & 2 \\ 0 & 1 & 0 & 1 & 3 \\ 0 & 1 & -1 & 1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & 4 & -1 & 2 \\ 0 & 1 & 0 & 1 & 3 \\ 0 & 0 & -1 & 0 & -3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & 4 & -1 & 2 \\ 0 & 1 & 0 & 1 & 3 \\ 0 & 0 & 1 & 0 & 3 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & -2 & 0 & -1 & -10 \\ 0 & 1 & 0 & 1 & 3 \\ 0 & 0 & 1 & 0 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 & -4 \\ 0 & 1 & 0 & 1 & 3 \\ 0 & 0 & 1 & 0 & 3 \end{bmatrix}$$

x_1, x_2, x_3 - Basic variables, x_4 - free variable

$$\begin{cases} x_4 = t \\ x_3 = 3 \\ x_2 = 3 - x_4 = 3 - t \\ x_1 = -4 - x_4 = -4 - t \end{cases} \Rightarrow x = \begin{pmatrix} -4 \\ 3 \\ 3 \\ 0 \end{pmatrix} + t \begin{pmatrix} -1 \\ -1 \\ 0 \\ 1 \end{pmatrix}, t \in \mathbb{R}.$$