July 19, 2007 **Quiz 5** Name: Solution Key

1. Let T be a linear transformation defined by $T(x) = Ax$ for $x \in R5$, where $A = \begin{pmatrix} 2 & -2 & 4 & 6 & 0 \\ 1 & 1 & 2 & 0 & -1 \\ 3 & 0 & -6 & 6 & -9 \end{pmatrix}$. Verify the dimension theorem for the transformation T.

Dimension Theorem: $\text{rank}(T) + \text{nullity}(T) = \dim(V)$, for $T: V \to W$.

or $\text{rank}(A) + \text{nullity}(A) = \dim(R^5) = 5$.

Find rank and nullity of A.

$$A = \begin{pmatrix} 2 & -2 & 4 & 6 & 0 \\ 1 & 1 & 2 & 0 & -1 \\ 3 & 0 & -6 & 6 & -9 \end{pmatrix} \to \begin{pmatrix} 1 & -1 & 2 & 3 & 0 \\ 1 & 1 & 2 & 0 & -1 \\ 1 & 0 & -2 & 2 & -3 \end{pmatrix} \to \begin{pmatrix} 1 & -1 & 2 & 3 & 0 \\ 0 & 2 & 0 & -3 & -1 \\ 0 & 1 & -4 & -1 & -3 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & -1 & 2 & 3 & 0 \\ 0 & 1 & -4 & -1 & -3 \\ 0 & 2 & 0 & -3 & -1 \end{pmatrix} \to \begin{pmatrix} 1 & -1 & 2 & 3 & 0 \\ 0 & 1 & -4 & -1 & -3 \\ 0 & 0 & 8 & * & * \end{pmatrix}$$

nullity$(A) = 2 = \#$ of nonpivot columns

rank $(A) = 3 = \#$ of pivot columns.

$2 + 3 = 5$.

2. Let $B = \left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 7 \end{bmatrix} \right\}$ and $C = \left\{ \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \end{bmatrix} \right\}$ be bases for R^2. Find the change of coordinate matrix from B-basis to C-basis and from C-basis to B-basis. Find the B-coordinate vector of vector $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Note that the coordinates of all vectors are given in the standard basis.

$$P_{E \leftarrow B} = \begin{pmatrix} 1 & 2 \\ 3 & 7 \end{pmatrix}, \quad P_{E \leftarrow C} = \begin{pmatrix} 2 & 3 \\ -1 & -1 \end{pmatrix}$$

$$P_{B \leftarrow E} = P_{E \leftarrow B}^{-1} = \begin{pmatrix} 1 & 2 \\ 3 & 7 \end{pmatrix}^{-1} = \frac{1}{7-6} \begin{pmatrix} 7 & -2 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} 7 & -2 \\ -3 & 1 \end{pmatrix}.$$

$$P_{B \leftarrow C} = P_{B \leftarrow E} P_{E \leftarrow C} = \begin{pmatrix} 7 & -2 \\ -3 & 1 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ -1 & -1 \end{pmatrix} = \boxed{\begin{pmatrix} 16 & 23 \\ -7 & -10 \end{pmatrix} \text{ C-basis to B-basis}}$$

$$P_{C \leftarrow B} = P_{B \leftarrow C}^{-1} = \frac{1}{-160+161} \begin{pmatrix} -10 & -23 \\ 7 & 16 \end{pmatrix} = \boxed{\begin{pmatrix} -10 & -23 \\ 7 & 16 \end{pmatrix} \text{ B-basis to C-basis}}$$

$$[x]_B = P_{B \leftarrow E} [x]_E = \begin{pmatrix} 7 & -2 \\ -3 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \boxed{\begin{pmatrix} 5 \\ -2 \end{pmatrix}}$$