July 16, 2007 **Quiz 4** Name: Solution Key

1. Let $A = \begin{pmatrix} 1 & 2 & -1 & 0 & 4 \\ 2 & 4 & 0 & 4 & 2 \\ -3 & -6 & 3 & 2 & -10 \end{pmatrix}$.

(a) Find a basis for the null space of matrix A. Solve $Ax=0$. Row Reduce A.

$$\to \begin{pmatrix} 1 & 2 & -1 & 0 & 4 \\ 0 & 0 & 2 & 4 & -6 \\ 0 & 0 & 0 & 2 & 2 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & -1 & 0 & 4 \\ 0 & 0 & 1 & 2 & -3 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & -1 & 0 & 4 \\ 0 & 0 & 1 & 0 & -5 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -5 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

x_2, x_5 - free variables.

$$\begin{cases} x_5 = t \\ x_4 = -t \\ x_3 = 5t \\ x_2 = s \\ x_1 = -2s + t \end{cases} \Rightarrow x = s\begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} 1 \\ 0 \\ 5 \\ -1 \\ 1 \end{pmatrix}.$$

Basis for null(A) is

$$\left\{ \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 5 \\ -1 \\ 1 \end{pmatrix} \right\}.$$

(b) Find a basis for the column space of matrix A. Pivot columns of A is a basis for Col(A).

$$\left\{ \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix} \right\}.$$

(c) What is the nullity of matrix A?

$\text{nullity}(A) = \dim(\text{Null}(A)) = 2.$

(d) What is rank of matrix A?

$\text{rank}(A) = n - \text{nullity}(A) = 5 - 2 = 3.$

2. Prove that the set $R^{2\times 2}_{sym}$ of symmetric 2×2 matrices is a subspace of $R^{2\times 2}$.

1) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \in R^{2\times 2}_{sym}$.

2) $\begin{pmatrix} a_1 & b_1 \\ b_1 & c_1 \end{pmatrix} + \begin{pmatrix} a_2 & b_2 \\ b_2 & c_2 \end{pmatrix} = \begin{pmatrix} a_1+a_2 & b_1+b_2 \\ b_1+b_2 & c_1+c_2 \end{pmatrix} = \begin{pmatrix} a_1' & b_1' \\ b_1' & c_1' \end{pmatrix} \in R^{2\times 2}_{sym}$ ← symmetric.

3) $\alpha \begin{pmatrix} a & b \\ b & c \end{pmatrix} = \begin{pmatrix} \alpha a & \alpha b \\ \alpha b & \alpha c \end{pmatrix} = \begin{pmatrix} a' & b' \\ b' & c' \end{pmatrix} \in R^{2\times 2}_{sym}$ (symmetric matrix).

$\Rightarrow R^{2\times 2}_{sym}$ is a subspace of $R^{2\times 2}$.

3. Determine if the set $\{2+t, 1+2t, 1-2t\}$ is linearly independent in P_1.

$\dim(P_1) = 2$. The set contains 3 polynomials $> \dim(P_1)$

$\Rightarrow$ the set is linearly dependent.