Quiz 3

Name: Solution Key

1. Find the value for c such that the following determinant is zero:

$$\begin{vmatrix} 1 & 3 & -2 & 4 \\ 2 & -1 & 2 & -4 \\ 3 & 2 & 0 & 2 \\ -1 & 2 & c & 10 \end{vmatrix} = \begin{vmatrix} 1 & 3 & -2 & 4 \\ 3 & 2 & 0 & 0 \\ 3 & 2 & 0 & 2 \\ -1 & 2 & c & 10 \end{vmatrix} R_2+R_1 = \begin{vmatrix} 1 & 3 & -2 & 4 \\ 3 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ -1 & 2 & c & 10 \end{vmatrix} R_3-R_2$$

$$= 2\cdot(-1)^{3+4}\begin{vmatrix} 1 & 3 & -2 \\ 3 & 2 & 0 \\ -1 & 2 & c \end{vmatrix} = -2\left(-2(-1)^{1+3}\begin{vmatrix} 3 & 2 \\ -1 & 2 \end{vmatrix} + c(-1)^{3+3}\begin{vmatrix} 1 & 3 \\ 3 & 2 \end{vmatrix}\right)$$

$$= -2\left(-2(6+2) + c(2-9)\right) = -2\left(-16-7c\right) = 0$$

$$c = -\frac{16}{7}.$$

2. A linear transformation T maps the vectors $e_1 = (1,0,0)^T$, $e_2 = (0,1,0)^T$, and $e_3 = (0,0,1)^T$ to the vectors $v_1 = (1,1,1)^T$, $v_2 = (0,2,1)^T$, and $v_3 = (1,1,2)^T$, respectively. Find the volume of the region $T(D)$, where D is a region in R^3 with volume π.

$$T: R^3 \to R^3, \quad T(x) = Ax, \ \forall x \in R^3, \quad A = [T(e_1), T(e_2), T(e_3)]$$

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}.$$

$$\text{Volume}(T(D)) = |\det(A)|\,\text{Volume}(D)$$

$$= \left|\begin{vmatrix} 1 & 0 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix}\right| \cdot \pi = \left|\begin{vmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{vmatrix}\right| \pi = 2\pi.$$