Quiz 2

Name: Solution Key

1. Let

$$S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix} \right\} \text{ and } u = \begin{bmatrix} -5 \\ 5 \\ -4 \end{bmatrix}$$

To answer questions in parts (a) and (c), we row reduce the corresponding augmented matrix

(a) Determine whether the set S is linearly independent or linearly dependent.

$$\begin{pmatrix} 1 & 2 & 4 & -5 \\ 2 & -1 & 3 & 5 \\ 3 & 4 & 5 & -4 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 4 & -5 \\ 0 & -5 & -5 & 15 \\ 0 & -2 & -7 & 11 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 4 & -5 \\ 0 & 1 & 1 & -3 \\ 0 & -2 & -7 & 11 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 4 & -5 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & -5 & 5 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 2 & 4 & -5 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 1 & -1 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{pmatrix}.$$

The set S is linearly independent since columns 1, 2, 3 are pivot columns

(b) Does the set S span R^3 (explain)?

Yes, it does. The linear system $Ax = u$ has a solution for any $u \in R^3$.

(c) Write vector u as a linear combination of vectors in S if possible.

$$u = 3v_1 - 2v_2 - v_3$$

2. Determine whether the transformation T is linear if

$T(x, y, z) = (2x - 3y, 4y - 5z, -6x + 2y)$.

$$T(x_1+x_2, y_1+y_2, z_1+z_2) = (2(x_1+x_2) - 3(y_1+y_2), 4(y_1+y_2) - 5(z_1+z_2), -6(x_1+x_2) + 2(y_1+y_2))$$
$$= (2x_1 - 3y_1, 4y_1 - 5z_1, -6x_1 + 2y_1) + (2x_2 - 3y_2, 4y_2 - 5z_2, -6x_2 + 2y_2)$$
$$= T(x_1, y_1, z_1) + T(x_2, y_2, z_2). \checkmark$$
$$T(cx, cy, cz) = (2cx - 3cy, 4cy - 5cz, -6cx + 2cy) = c(2x - 3y, 4y - 5z, -6x + 2y)$$
$$= cT(x, y, z) \checkmark$$

T is a linear transformation.

3. Find the standard matrix of the linear transformation $T : R^2 \to R^2$ that first reflects points through the line $x_2 = -x_1$, then rotates through the angle $\pi/2$.

$$T = \underbrace{T_2}_{\text{rotation}} \underbrace{T_1}_{\text{reflection}}$$

$$T_1(e_1) = \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \quad T_1(e_2) = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$T_2\begin{pmatrix} 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad T_2\begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$\boxed{[T] = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}$$