June 18, 2006 **Quiz 1** **Name:** Solution Key

1. Find all solutions of the linear system.

(a)
$$\begin{aligned} 2x + y - 4z &= 8 \\ 3x - y + 2z &= -1 \\ x - 3y - 10z &= 17 \end{aligned}$$

$$[A|b] = \begin{bmatrix} 2 & 1 & -4 & 8 \\ 3 & -1 & 2 & -1 \\ 1 & -3 & -10 & 17 \end{bmatrix} \to \begin{bmatrix} 1 & -3 & -10 & 17 \\ 3 & -1 & 2 & -1 \\ 2 & 1 & -4 & 8 \end{bmatrix} \begin{matrix} R_3 \\ R_2 \\ R_1 \end{matrix}$$

$$\to \begin{bmatrix} 1 & -3 & -10 & 17 \\ 0 & 8 & 32 & -52 \\ 0 & 7 & 16 & -26 \end{bmatrix} \begin{matrix} \\ R_2 - 3R_1 \\ R_3 - 2R_1 \end{matrix} \to \begin{bmatrix} 1 & -3 & -10 & 17 \\ 0 & 1 & 16 & -26 \\ 0 & 7 & 16 & -26 \end{bmatrix} R_2 - R_3$$

$$\to \begin{bmatrix} 1 & -3 & -10 & 17 \\ 0 & 1 & 16 & -26 \\ 0 & 6 & 0 & 0 \end{bmatrix} R_3 - R_2 \to \begin{bmatrix} 1 & -3 & -10 & 17 \\ 0 & 1 & 16 & -26 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tfrac{1}{6}R_3 \to \begin{bmatrix} 1 & -3 & -10 & 17 \\ 0 & 1 & 16 & -26 \\ 0 & 0 & -16 & 26 \end{bmatrix} R_3 - R_2$$

$$\to \begin{bmatrix} 1 & -3 & -10 & 17 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -16 & 26 \end{bmatrix} R_2 + R_3 \to \begin{bmatrix} 1 & -3 & -10 & 17 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -13/8 \end{bmatrix} \to \begin{bmatrix} 1 & -3 & 0 & 3/4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -13/8 \end{bmatrix} R_1 + 10R_3$$

$$\to \begin{bmatrix} 1 & 0 & 0 & 3/4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -13/8 \end{bmatrix} \Rightarrow \boxed{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3/4 \\ 0 \\ -13/8 \end{pmatrix}.}$$

(b)
$$\begin{aligned} x \quad + y \quad + \quad 2z &= 1 \\ -2x \qquad\qquad - z &= 2 \\ x \quad + 3y \quad + \quad 5z &= 2 \end{aligned}$$

$$[A|b] = \begin{bmatrix} 1 & 1 & 2 & 1 \\ -2 & 0 & -1 & 2 \\ 1 & 3 & 5 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & 2 & 3 & 4 \\ 0 & 2 & 3 & 1 \end{bmatrix} \begin{matrix} \\ R_2 + 2R_1 \\ R_3 - R_1 \end{matrix} \to \begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & 2 & 3 & 4 \\ 0 & 0 & 0 & -3 \end{bmatrix} R_3 - R_2$$

$\boxed{\text{System is inconsistent}}$ since $0 \cdot x_3 \neq -3$.

2. Write the vector $b = \begin{pmatrix} 9 \\ -12 \end{pmatrix}$ as a linear combination of the vectors $a_1 = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$ and $a_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$. Is such linear combination unique? Explain.

$$c_1 \begin{pmatrix} 2 \\ -2 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 9 \\ -12 \end{pmatrix} \Rightarrow \begin{pmatrix} 2 & 1 & 9 \\ -2 & 2 & -12 \end{pmatrix} \to \begin{pmatrix} 1 & -1 & 6 \\ 2 & 1 & 9 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & -1 & 6 \\ 0 & 3 & -3 \end{pmatrix} \to \begin{pmatrix} 1 & -1 & 6 \\ 0 & 1 & -1 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 5 \\ 0 & 1 & -1 \end{pmatrix}$$

x_1, x_2 - basic variables.

$\boxed{x_1 = 5, \quad x_2 = -1}$