November 15, 2007 **Quiz 6** **Name:** Solution Key

Diagonalize the matrix if possible:

1. $A = \begin{pmatrix} 2 & 5 \\ 0 & 2 \end{pmatrix}$

$P_A(t) = \det(A - tI_2) = \begin{vmatrix} 2-t & 5 \\ 0 & 2-t \end{vmatrix} = (2-t)^2 = 0 \Rightarrow \lambda_1 = 2, m_1 = 2$

let's determine n_1.

$A - 2I_2 = \begin{pmatrix} 0 & 5 \\ 0 & 0 \end{pmatrix} \to \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \Rightarrow n_1 = 1 < m_1 \Rightarrow$

matrix isn't diagonalizable.

2. $A = \begin{pmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}$ $P_A(t) = \begin{vmatrix} 3-t & -2 & 0 \\ -2 & 3-t & 0 \\ 0 & 0 & 5-t \end{vmatrix} = (5-t)\begin{vmatrix} 3-t & -2 \\ -2 & 3-t \end{vmatrix}$

$= (5-t)\left[(3-t)^2 - 4\right] = (5-t)(3-t-2)(3-t+2)$

$= (5-t)^2(1-t) = 0$

$\Rightarrow \lambda_1 = 1, m_1 = 1$

$\lambda_2 = 5, m_2 = 2$

$A - \lambda_1 I_3 = A - I_3 = \begin{pmatrix} 2 & -2 & 0 \\ -2 & 2 & 0 \\ 0 & 0 & 4 \end{pmatrix} \to \begin{pmatrix} 1 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

$\Rightarrow n_1 = 1$, $\begin{matrix} x_3 = 0 \\ x_2 = t \\ x_1 = t \end{matrix} \Rightarrow x = t\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \Rightarrow v_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

$A - \lambda_2 I_3 = A - 5I_3 = \begin{pmatrix} -2 & -2 & 0 \\ -2 & -2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow n_2 = 2$

$n_1 + n_2 = 1 + 2 = 3 = n \Rightarrow$ <u>A is diagonalizable</u>.

$\begin{cases} x_3 = t \\ x_2 = s \\ x_1 = -s \end{cases} \Rightarrow x = s\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \Rightarrow v_2 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, v_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$\Rightarrow A = VDV^{-1}$, where $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ and $V = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.