October 30, 2007 **Quiz 5** Name: Solution Key

1. Let

$$A = \begin{pmatrix} 1 & -1 & 2 & 3 & -2 \\ 2 & 4 & -2 & 0 & 2 \\ 1 & 3 & -1 & 4 & 0 \end{pmatrix}.$$

(a) Find a basis for the column space of the matrix.

Row reduce A and identify pivot columns of A.

$$A \to \begin{pmatrix} 1 & -1 & 2 & 3 & -2 \\ 0 & 6 & -6 & -6 & 6 \\ 0 & 4 & -3 & 1 & 2 \end{pmatrix} \to \begin{pmatrix} 1 & -1 & 2 & 3 & -2 \\ 0 & 1 & -1 & -1 & 1 \\ 0 & 4 & -3 & 1 & 2 \end{pmatrix} \to \underbrace{\begin{pmatrix} 1 & -1 & 2 & 3 & -2 \\ 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 1 & 5 & -2 \end{pmatrix}}_{REF}$$

Note, columns 1, 2, and 3 are pivot $\Rightarrow$

Basis for $col(A)$ is $\left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix} \right\}$.

(b) Find the basis for the row space of the matrix.

Basis for the rowspace is

$\{(1,-1,2,3,-2), (0,1,-1,-1,1), (0,0,1,5,-2)\}$.

(c) Find the basis for the null space of the matrix. Find the general solution of $Ax=0$.

Nee RREF of A.

$$\begin{pmatrix} 1 & -1 & 2 & 3 & -2 \\ 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 1 & 5 & -2 \end{pmatrix} \to \begin{pmatrix} 1 & -1 & 0 & -7 & 2 \\ 0 & 1 & 0 & 4 & -1 \\ 0 & 0 & 1 & 5 & -2 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 & -3 & 1 \\ 0 & 1 & 0 & 4 & -1 \\ 0 & 0 & 1 & 5 & -2 \end{pmatrix}$$

x_1, x_2, x_3 – basic vars.
x_4, x_5 – free vars.

$x_5 = s$, $x_4 = t$

$x_3 = -5x_4 + 2x_5 = -5t + 2s$,

$x_2 = -4x_4 + x_5 = -4t + s$,

$x_1 = 3x_4 - x_5 = 3t - s$

$$\Rightarrow x = s\begin{pmatrix} -1 \\ 1 \\ 2 \\ 0 \\ 1 \end{pmatrix} + t\begin{pmatrix} 3 \\ -4 \\ -5 \\ 1 \\ 0 \end{pmatrix}.$$

Basis for $Null(A)$

$$\left\{ \begin{pmatrix} -1 \\ 1 \\ 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ -4 \\ -5 \\ 1 \\ 0 \end{pmatrix} \right\}.$$

(d) Determine the nullity and the rank of the matrix.

nullity$(A) = 2$, rank$(A) = 3$.

2. Consider the vector space P_2 with the basis $\mathcal{B} = \{1+t+t^2, 2-t+t^2, -t+t^2\}$. Find the $\mathcal{B}$-coordinate vector of the polynomial $p(t) = -4 + 9t - 5t^2$.

$$P_{E \leftarrow B} = \begin{pmatrix} 1 & 2 & 0 \\ 1 & -1 & -1 \\ 1 & 1 & 1 \end{pmatrix}. \quad [P]_E = \begin{pmatrix} -4 \\ 9 \\ -5 \end{pmatrix}, [P]_E = P_{E \leftarrow B}[P]_B.$$

Solve $\begin{pmatrix} 1 & 2 & 0 \\ 1 & -1 & -1 \\ 1 & 1 & 1 \end{pmatrix} x = \begin{pmatrix} -4 \\ 9 \\ -5 \end{pmatrix}$.

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & -4 \\ 1 & -1 & -1 & 9 \\ 1 & 1 & 1 & -5 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 2 & 0 & -4 \\ 0 & -3 & -1 & 13 \\ 0 & -1 & 1 & -1 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 2 & 0 & -4 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & -4 & 16 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|c} 1 & 2 & 0 & -4 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -4 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 2 & 0 & -4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -4 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -4 \end{array}\right]$$

$$\Rightarrow [p]_B = \begin{pmatrix} 2 \\ -3 \\ -4 \end{pmatrix}.$$