Quiz 4

Name: Solution Key

1. Find the value of the determinant (Hint: use the elementary column operations and the entry a_{21} as a pivot):

$$D = \begin{vmatrix} 1 & 2 & 2 & 3 \\ 1 & 0 & 2 & 0 \\ 3 & -1 & 1 & -2 \\ 1 & -2 & 0 & 3 \end{vmatrix}.$$

1) Eliminate entry a_{23}: $-2\cdot C_1 + C_3 \to C_3$

$$D = \begin{vmatrix} 1 & 2 & 0 & 3 \\ 1 & 0 & 0 & 0 \\ 3 & -1 & -5 & -2 \\ 1 & -2 & -2 & 3 \end{vmatrix}$$

2) Use cofactor expansion across 2nd Row:

$$D = (-1)^{2+1}\cdot 1 \begin{vmatrix} 2 & 0 & 3 \\ -1 & -5 & -2 \\ -2 & -2 & 3 \end{vmatrix} = \begin{vmatrix} 2 & 0 & 3 \\ 1 & 5 & 2 \\ -2 & -2 & 3 \end{vmatrix} = \begin{vmatrix} 0 & -10 & -1 \\ 1 & 5 & 2 \\ 0 & 8 & 7 \end{vmatrix} \begin{matrix} R_1 - 2R_2 \\ \\ R_3 + 2R_2 \end{matrix}$$

$$= (-1)^{2+1}\cdot 1 \begin{vmatrix} -10 & -1 \\ 8 & 7 \end{vmatrix} = -(-70+8) = 62$$

2. A linear transformation $T : R^3 \to R^3$ maps the vectors $e_1 = (1,0,0)^T$, $e_2 = (0,1,0)^T$, and $e_3 = (0,0,1)^T$ to the vectors $v_1 = (1,2,1)^T$, $v_2 = (0,2,-1)^T$, and $v_3 = (0,2,3)^T$, respectively. Find the volume of the region $T(D)$, where D is an ellipsoid defined by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

Hint: The volume of the ellipsoid is $(4/3)\pi abc$.

$$\text{Volume}(T(D)) = \pm \begin{vmatrix} 1 & 0 & 0 \\ 2 & 2 & 2 \\ 1 & -1 & 3 \end{vmatrix} \text{Volume}(D)$$

$$= \pm \begin{vmatrix} 2 & 2 \\ -1 & 3 \end{vmatrix} \frac{4}{3}\pi abc = \frac{8\cdot 4}{3}\pi abc = \frac{32}{3}\pi abc.$$