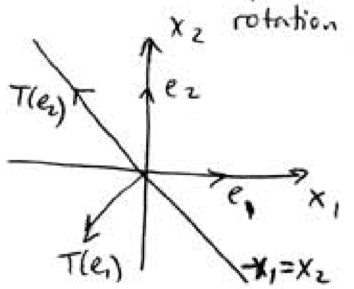
Quiz 3 Name: Solution Key

1. Let $T : R^2 \to R^2$ be the linear transformation that, first, rotates counter-clockwise through the angle and then reflects with respect the line $x_1 + x_2 = 0$. Find the standard matrix of transformation T.

Apply T on $e_1 = \begin{pmatrix}1\\0\end{pmatrix}$ and $e_2 = \begin{pmatrix}0\\1\end{pmatrix}$. Then $[T] = [T(e_1), T(e_2)]$.

$$e_1 = \begin{pmatrix}1\\0\end{pmatrix} \underset{\text{rotation}}{\to} \begin{pmatrix}\cos \pi/4\\ \sin \pi/4\end{pmatrix} \underset{\text{reflection.}}{\to} \begin{pmatrix}-\sqrt{2}/2\\ -\sqrt{2}/2\end{pmatrix}$$



$$e_2 = \begin{pmatrix}0\\1\end{pmatrix} \underset{\text{rotation}}{\to} \begin{pmatrix}-\sqrt{2}/2\\ \sqrt{2}/2\end{pmatrix} \underset{\text{reflection.}}{\to} \begin{pmatrix}-\sqrt{2}/2\\ \sqrt{2}/2\end{pmatrix}$$

$$[T] = \begin{bmatrix}-\sqrt{2}/2 & -\sqrt{2}/2\\ -\sqrt{2}/2 & \sqrt{2}/2\end{bmatrix}.$$

2. Determine if the function $T : R^3 \to R^2$ defined by $T(x_1, x_2, x_3) = (2x_1 - x_2 + 5x_3,\ x_1 + x_2 - 3)$ is a li transformation. Explain.

If $T(0) \neq 0$, then T is not a linear transformation.

$T(0,0,0) = (2\cdot 0 - 0 + 5\cdot 0,\ 0 + 0 - 3) = (0, -3) \neq (0,0) \Rightarrow T$ is NOT a lin. trans

Alternative solution: Check the definition of a lin. transformation.

1) $T(\alpha x) = \alpha T(x)$?

$T(\alpha x) = T(\alpha x_1, \alpha x_2, \alpha x_3) = (2(\alpha x_1) - (\alpha x_2) + 5(\alpha x_3),\ \alpha x_1 + (\alpha x_2) - 3)$

$= (\alpha(2x_1 - x_2 + 5x_3, \alpha(x_1 + x_2 - \frac{3}{\alpha}) = \alpha(2x_1 - x_2 + 5x_3,\ x_1 + x_2 - \frac{3}{\alpha}))$

$\neq \alpha T(x)$, for any $\alpha \neq 1$.

3. A linear transformation $T : R^3 \to R^2$ has the following standard matrix $[T]$. Determine if the transforma T is onto, one-to-one, or a bijection. Explain.

$$[T] = \begin{pmatrix}1 & 3 & -2\\ 2 & 6 & -5\end{pmatrix},$$

$[T]$ is 2×3. T is NOT one-to-one

T is NOT a bijection.

Row reduce $[T]$ to see if T is ONTO.

$$[T] = \begin{bmatrix}1 & 3 & -2\\ 2 & 6 & -5\end{bmatrix} \to \begin{bmatrix}1 & 3 & -2\\ 0 & 0 & 1\end{bmatrix}$$ REF. $\exists$ pivot in every row.

$\Rightarrow T$ is ONTO