Quiz 2 **Name:** Solution Key

1. Let

$$S = \left\{ \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ -2 \\ 1 \end{bmatrix} \right\} \text{ and } b = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

(v_1, v_2, v_3)

(a) Determine if vector b belongs to the span of vectors in S?

Check if the linear system $[v_1, v_2, v_3]x = b$ has a solution.

$$[A|b] = \begin{bmatrix} 2 & 1 & -3 & 1 \\ 3 & 2 & -2 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 1 & 1 \\ 3 & 2 & -2 & 2 \\ 2 & 1 & -3 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & -5 & -1 \\ 0 & -1 & -5 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 5 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ REF}$$

System $Ax = b$ is consistent.

Therefore, b belongs to the span$\{v_1, v_2, v_3\}$.

Note that $b = v_2$. Therefor $b \in$ span$\{v_1, v_2, v_3\}$

(b) Is the set S linearly independent or linearly dependent? Explain.

Let $A = [v_1, v_2, v_3]$. The set is linearly independent if all columns of matrix A are pivot columns.

REF of A is $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{bmatrix}$ (see part (a)). Column 3 is not pivot.

Set S is linearly dependent.

(c) Does the set S span R^3? Explain.

Set S spans R^3 if A has a pivot in every row.

Row 3 doesn't have a pivot. Therefore, set S does NOT span R^3.

2. Determine if the linear system is consistent. If it is consistent, find the general solution of the system in a vector-parametric form.

$$\begin{aligned} x_1 + 3x_2 - 2x_3 + 2x_5 &= 0 \\ 2x_1 + 6x_2 - 5x_3 - 2x_4 + 4x_5 - 3x_6 &= -1 \\ 5x_3 + 10x_4 + 15x_6 &= 5 \end{aligned}$$

There are 3 equations with 6 unknowns.

Coefficient matrix is 3×6.

$[A|b]$ is 3×7

$$\left[\begin{array}{cccccc|c} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 2 & 6 & -5 & -2 & 4 & -3 & -1 \\ 0 & 0 & 5 & 10 & 0 & 15 & 5 \end{array}\right] \to \left[\begin{array}{cccccc|c} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & -1 & -2 & 0 & -3 & -1 \\ 0 & 0 & 1 & 2 & 0 & 3 & 1 \end{array}\right] \begin{matrix} \\ R_2 - 2R_1 \\ \frac{1}{5}R_3 \end{matrix}$$

$$\to \left[\begin{array}{cccccc|c} \textcircled{1} & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & \textcircled{1} & 2 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

System is consistent. Obtain RREF.

Columns 1 and 3 are pivot

x_1, x_3 – basic variables.

x_2, x_4, x_5, x_6 – free variables.

$$\to \left[\begin{array}{cccccc|c} \textcircled{1} & 3 & 0 & 4 & 2 & 6 & 2 \\ 0 & 0 & \textcircled{1} & 2 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{cases} x_6 = s_1 \\ x_5 = s_2 \\ x_4 = s_3 \\ x_3 = 1 - 2s_3 - 3s_1 \\ x_2 = s_4 \\ x_1 = 2 - 3s_4 - 4s_3 - 2s_2 - 6s_1 \end{cases}$$

$$x = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + s_1 \begin{bmatrix} -6 \\ 0 \\ -3 \\ 0 \\ 0 \\ 1 \end{bmatrix} + s_2 \begin{bmatrix} -2 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} + s_3 \begin{bmatrix} -4 \\ 0 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s_4 \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad s_i \in R \quad i = 1,3,5,4.$$

$\leftarrow$ solution.