August 30, 2007 **Quiz 1** Name: Solution Key

1. Determine if the linear system is consistent.

(a)
$$\begin{array}{rrrrl} -2x_1 & +2x_2 & & -6x_4 & = 4 \\ x_1 & -x_2 & +2x_3 & +3x_3 & = -4 \\ 4x_1 & +2x_2 & -4x_3 & -6x_4 & = -4 \end{array}$$

Row reduce the augmented matrix and see if there are rows of the form $[0,\dots,0|\beta_{\neq 0}]$. If yes, then the system is inconsistent.

$$\begin{bmatrix} -2 & 2 & 0 & 1 & 4 \\ 1 & -1 & 2 & 3 & 4 \\ 4 & 2 & -2 & 1 & 5 \end{bmatrix} \to \begin{bmatrix} 1 & -1 & 2 & 3 & 4 \\ -2 & 2 & 0 & 1 & 4 \\ 4 & 2 & -2 & 1 & 5 \end{bmatrix} \to \begin{bmatrix} 1 & -1 & 2 & 3 & 4 \\ 0 & 0 & 4 & 7 & 12 \\ 0 & 6 & -10 & -11 & -11 \end{bmatrix}$$

$$\to \underbrace{\begin{bmatrix} 1 & -1 & 2 & 3 & 4 \\ 0 & 6 & -10 & -11 & -11 \\ 0 & 0 & 4 & 7 & 12 \end{bmatrix}}_{\text{REF}}.$$

The system is consistent since there are now rows of the form $[0,0,0,0,\beta]$, $\beta \neq 0$.

(b)
$$\begin{array}{rrrl} x & +2y & +2z = & 3 \\ -2x & +2y & -z = & 6 \\ x & +8y & +5z = & 16 \end{array}$$

Same solution as in part a).

$$\begin{bmatrix} 1 & 2 & 2 & 3 \\ -2 & 2 & -1 & 6 \\ 1 & 8 & 5 & 16 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 2 & 3 \\ 0 & 6 & 3 & 12 \\ 0 & 6 & 3 & 13 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 2 & 3 \\ 0 & 6 & 3 & 12 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{REF}$$

The system is inconsistent. The last row is equivalent to the equation $0 \cdot z = 1$, which has no solutions.

2. Determine if vector b belongs to the span of vectors v_1, v_2, and v_3:

$$b = \begin{pmatrix} 4 \\ 2 \\ 6 \end{pmatrix}, \quad v_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad v_2 = \begin{pmatrix} -1 \\ -2 \\ -3 \end{pmatrix}, v_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}.$$

Row reduce $[v_1, v_2, v_3, b]$ and see if there is a row $[0,0,0,\beta_{\neq 0}]$. If yes, then $b \notin \operatorname{span}\{v_1, v_2, v_3\}$.

$$\begin{pmatrix} 1 & -1 & 1 & 4 \\ 2 & -2 & 0 & 2 \\ 3 & -3 & 1 & 6 \end{pmatrix} \to \begin{pmatrix} 1 & -1 & 1 & 4 \\ 0 & 0 & -2 & -6 \\ 0 & 0 & -2 & -6 \end{pmatrix} \to \begin{pmatrix} 1 & -1 & 1 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix} \text{REF}$$

The vector equation $c_1 v_1 + c_2 v_2 + c_3 v_3 = b$ is consistent.
Therefore, b belongs to the span $\{v_1, v_2, v_3\}$.