November 7, 2006 **Quiz 5** Name: Solution Key

1. Let

$$A = \begin{pmatrix} 1 & 3 & -2 & 5 & 4 \\ 1 & 4 & 1 & 3 & 5 \\ 1 & 4 & 2 & 4 & 3 \end{pmatrix} \to \begin{pmatrix} 1 & 3 & -2 & 5 & 4 \\ 0 & 1 & 3 & -2 & 1 \\ 0 & 1 & 4 & -1 & -1 \end{pmatrix} \to \begin{pmatrix} 1 & 3 & -2 & 5 & 4 \\ 0 & 1 & 3 & -2 & 1 \\ 0 & 0 & 1 & 1 & -2 \end{pmatrix} \leftarrow \text{REF of A}$$

(a) Find a basis for the column space of the matrix.

1st, 2nd, and 3rd columns of A are pivot =>

$\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 4 \\ 4 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix} \right\}$ is a basis for Col (A)

(b) Find the basis for the row space of the matrix.

$\{(1,3,-2,5,4), (0,1,3,-2,1), (0,0,1,1,-2)\}$ is a basis for Row (A)

(c) Find the basis for the null space of the matrix.

$$\begin{pmatrix} 1 & 3 & -2 & 5 & 4 \\ 0 & 1 & 3 & -2 & 1 \\ 0 & 0 & 1 & 1 & -2 \end{pmatrix} \to \begin{pmatrix} 1 & 3 & 0 & 7 & 0 \\ 0 & 1 & 0 & -5 & 7 \\ 0 & 0 & 1 & 1 & -2 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 & 22 & -21 \\ 0 & 1 & 0 & -5 & 7 \\ 0 & 0 & 1 & 1 & -2 \end{pmatrix} \leftarrow \text{RREF of A}$$

x_1, x_2, x_3 - basic variables; x_4, x_5 - free variables.

$x_5 = t$

$x_4 = s$

$x_3 = -x_4 + 2x_5 = -s + 2t$

$x_2 = 5x_4 - 7x_5 = 5s - 7t$,

$x_1 = -22x_4 + 21x_5 = -22s + 21t$,

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = s\begin{pmatrix} -22 \\ 5 \\ -1 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 21 \\ -7 \\ 2 \\ 0 \\ 1 \end{pmatrix}, \quad \left\{ \begin{pmatrix} -22 \\ 5 \\ -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 21 \\ -7 \\ 2 \\ 0 \\ 1 \end{pmatrix} \right\}$$

basis for Null (A).

(d) Determine the nullity (the dimension of the nullspace) and the rank of the matrix.

nullity (A) = 2, rank (A) = 3.

2. Consider the vector space P_2 with the basis $\mathcal{B} = \{1, t+1, t^2+t\}$. Find the $\mathcal{B}$-coordinate vector of the polynomial $p(t) = -t^2 - 2t + 3$.

Let $C = \{1, t, t^2\}$. Then $[p]_C = \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$, $P_{C \leftarrow B} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

$P_{C\leftarrow B}[p]_B = [p]_C \Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}[p]_c = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$. Let $[p]_c = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}$.

$$\begin{cases} p_1 + p_2 = 3 \\ p_2 + p_3 = -2 \\ p_3 = -1 \end{cases} \Rightarrow \begin{cases} p_3 = -1 \\ p_2 = -2 - p_3 = -2 + 1 = -1 \\ p_1 = 3 - p_2 = 3 + 1 = 4. \end{cases} \quad \boxed{[p]_B = \begin{bmatrix} 4 \\ -1 \\ -1 \end{bmatrix}}$$