November 7, 2006 **Quiz 5** Name: Solution Key

1. Let

$$A = \begin{pmatrix} 1 & 1 & 0 & -1 & 2 \\ 1 & 2 & 3 & 0 & 1 \\ 2 & 3 & 3 & -1 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 0 & -1 & 2 \\ 0 & 1 & 3 & 1 & -1 \\ 0 & 1 & 3 & 1 & -4 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 0 & -1 & 2 \\ 0 & 1 & 3 & 1 & -1 \\ 0 & 0 & 0 & 0 & -3 \end{pmatrix} \text{ REF of } A$$

(a) Find a basis for the column space of the matrix.

1st, 2nd, and 5th columns are pivot

$\left\{ \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \right\}$ is a basis for $Col(A)$.

(b) Find the basis for the row space of the matrix.

$\{(1,1,0,-1,2),(0,1,3,1,-1),(0,0,0,0,-3)\}$ – Basis for $Row(A)$.

(c) Find the basis for the null space of the matrix.

$$\begin{pmatrix} 1 & 1 & 0 & -1 & 2 \\ 0 & 1 & 3 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 0 & -1 & 0 \\ 0 & 1 & 3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & -3 & -2 & 0 \\ 0 & 1 & 3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \text{ RREF of } A$$

x_3, x_4 – free variables.

$x_5 = 0$

$x_4 = t$

$x_3 = s$

$x_2 = -3x_3 - x_4 = -3s - t$

$x_1 = 3x_3 + 2x_4 = 3s + 2t$

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = s\begin{pmatrix} 3 \\ -3 \\ 1 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} 2 \\ -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \boxed{\left\{ \begin{pmatrix} 3 \\ -3 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right\} \text{ – Basis for } Null(A)}$$

(d) Determine the nullity (the dimension of the nullspace) and the rank of the matrix.

$nullity(A) = 2$, $rank(A) = 3$.

2. Consider the vector space P_2 with the basis $\mathcal{B} = \{1, t+1, t^2+t\}$. Find the $\mathcal{B}$-coordinate vector of the polynomial $p(t) = t^2 - 3t + 2$. Let $C = \{1, t, t^2\}$.

$$P_{C\leftarrow B} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \quad [p]_C = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}, \quad P_{C\leftarrow B}[p]_B = [p]_C$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}, \quad \begin{cases} p_1 + p_2 = 2 \\ p_2 + p_3 = -3 \\ p_3 = 1 \end{cases} \Rightarrow \begin{cases} p_3 = 1 \\ p_2 = -3 - p_3 = -3 - 1 = -4 \\ p_1 = 2 - p_2 = 2 + 4 = 6. \end{cases} \quad \boxed{[p]_B = \begin{bmatrix} 6 \\ -4 \\ 1 \end{bmatrix}}$$