

October 18, 2006 Quiz 4

Name: Solution Key

1. Solve the linear system using the Cramer's rule

$$2x - 3y = -1$$

$$4x + 7y = -1$$

$$A = \begin{pmatrix} 2 & -3 \\ 4 & 7 \end{pmatrix}, A_1 = \begin{pmatrix} -1 & -3 \\ -1 & 7 \end{pmatrix}, A_2 = \begin{pmatrix} 2 & -1 \\ 4 & -1 \end{pmatrix}$$

$$|A| = 14 + 12 = 26, |A_1| = -7 - 3 = -10, |A_2| = -2 + 4 = 2$$

$$x = \frac{|A_1|}{|A|} = \frac{-10}{26} = -\frac{5}{13}$$

$$y = \frac{|A_2|}{|A|} = \frac{2}{26} = \frac{1}{13}$$

2. Find the volume of the parallelepiped determined by the vectors  $\begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ -4 \end{pmatrix}, \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$ .

$$\text{Volume} = \left| \det \begin{pmatrix} 1 & 1 & 4 \\ 1 & -2 & 1 \\ 3 & -4 & 2 \end{pmatrix} \right| = |-4 + 3 - 16 + 24 - 2 + 4| = 9$$

3. Let  $S$  be a bounded region on the plain with the area equal to 4, and let  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be a linear transformation defined by  $Tx = Ax, x \in \mathbb{R}^2$ , where  $A = \begin{pmatrix} 2 & 4 \\ -1 & 3 \end{pmatrix}$ . Find the area of the image of the region  $S$  under the transformation  $T$ .

$$\text{Area}(T(S)) = |\det(A)| \text{Area}(S) = \left| \det \begin{pmatrix} 2 & 4 \\ -1 & 3 \end{pmatrix} \right| \cdot 4 = |6 + 4| \cdot 4 = 40.$$