

October 18, 2006

## Quiz 4

Name: Solution Key

1. Solve the linear system using the Cramer's rule

$$\begin{aligned} 3x + 5y &= 8 \\ 4x - 2y &= 1 \end{aligned} \quad A = \begin{pmatrix} 3 & 5 \\ 4 & -2 \end{pmatrix}, \quad A_1 = \begin{pmatrix} 8 & 5 \\ 1 & -2 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 3 & 8 \\ 4 & 1 \end{pmatrix}$$

$$|A| = -6 - 20 = -26, \quad |A_1| = -16 - 5 = -21, \quad |A_2| = 3 - 32 = -29$$

$$x = \frac{|A_1|}{|A|} = \frac{-21}{-26} = \frac{21}{26}, \quad y = \frac{|A_2|}{|A|} = \frac{-29}{-26} = \frac{29}{26}$$

2. Find the volume of the parallelepiped determined by the vectors
- $\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}, \begin{pmatrix} 3 \\ 4 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$
- .

$$\text{Volume} = \text{abs} \begin{vmatrix} 1 & 3 & 2 \\ 2 & 4 & -1 \\ -3 & -1 & 5 \end{vmatrix} = |20 + 4 - 4 + 24 - 1 - 30| = 18$$

3. Let
- $S$
- be a bounded region on the plain with the area equal to 5, and let
- $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$
- be a linear transformation defined by
- $Tx = Ax, x \in \mathbb{R}^2$
- , where
- $A = \begin{pmatrix} 1 & -3 \\ 4 & -1 \end{pmatrix}$
- . Find the area of the image of the region
- $S$
- under the transformation
- $T$
- .

$$\text{area}(T(S)) = |\det(A)| \text{area}(S) = \left| \det \begin{pmatrix} 1 & -3 \\ 4 & -1 \end{pmatrix} \right| \cdot 5 = 55.$$