September 21, 2006 **Quiz 3** Name: Solution Key

1. Find the values of h for which the set of vectors is linearly dependent: $\left\{ \begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix}, \begin{bmatrix} -6 \\ 7 \\ -3 \end{bmatrix}, \begin{bmatrix} 8 \\ h \\ 4 \end{bmatrix} \right\}$

$$\begin{bmatrix} 2 & -6 & 8 \\ -4 & 7 & h \\ 1 & -3 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & -3 & 4 \\ -4 & 7 & h \\ 1 & -3 & 4 \end{bmatrix} \tfrac{1}{2}R_1 \to \begin{bmatrix} 1 & -3 & 4 \\ 0 & -5 & 16+h \\ 0 & 0 & 0 \end{bmatrix}$$

The vectors are linearly dependent for any $h \in \mathbb{R}$.

2. Find the standard matrix of a linear transformation $T : R^2 \to R^2$ that first contracts horizontally by a facto of 0.5 and then reflects points through the line $x_2 = x_1$.

Let $[T] \in \mathbb{R}^{2\times 2}$ be the matrix of T.

$[T] = [T(e_1), T(e_2)]$, where $e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$T(e_1) = \begin{bmatrix} 0 \\ 1/2 \end{bmatrix}$ since $\begin{bmatrix} 1 \\ 0 \end{bmatrix} \to \begin{bmatrix} 1/2 \\ 0 \end{bmatrix} \to \begin{bmatrix} 0 \\ \frac{1}{2} \end{bmatrix}$.

$T(e_2) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ since $\begin{bmatrix} 0 \\ 1 \end{bmatrix} \to \begin{bmatrix} 0 \\ 1 \end{bmatrix} \to \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Thus, $[T] = \begin{bmatrix} 0 & 1 \\ 1/2 & 0 \end{bmatrix}$

3. The linear transformation $T : R^3 \to R^2$ is such that $T(e_1) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, $T(e_2) = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$, $T(e_3) = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$ where e_i is the i standard basis vector in R^3 for $i = 1, 2, 3$. Is mapping T onto, one-to-one, or a bijection Explain.

$[T] = [T(e_1), T(e_2), T(e_3)] = \begin{bmatrix} 1 & 2 & -3 \\ -2 & -4 & 1 \end{bmatrix}$

T is not one-to-one since #columns > # rows.

T is not a bijection since it is not one-to-one.

$\begin{bmatrix} 1 & 2 & -3 \\ -2 & -4 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & -3 \\ 0 & 0 & -5 \end{bmatrix}$. There are pivot positions in all rows $\Rightarrow$ T is onto.