September 21, 2006 **Quiz 3** Name: Solution Key

1. Find the values of h for which the set of vectors is linearly dependent: $\left\{ \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix}, \begin{bmatrix} -5 \\ 7 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ h \end{bmatrix} \right\}$

$$\begin{bmatrix} 1 & -5 & 1 \\ -1 & 7 & 1 \\ -3 & 8 & h \end{bmatrix} \to \begin{bmatrix} 1 & -5 & 1 \\ 0 & 2 & 2 \\ 0 & -7 & 3+h \end{bmatrix} \to \begin{bmatrix} 1 & -5 & 1 \\ 0 & 1 & 1 \\ 0 & -7 & 3+h \end{bmatrix} \to \begin{bmatrix} 1 & -5 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 10+h \end{bmatrix}$$

The vectors are linearly dependent is there is a non-pivot column. If $h=-10$ the colum-3 is non-pivot.

For $h=-10$, the vectors are linearly dependent.

2. Find the standard matrix of a linear transformation $T : R^2 \to R^2$ that first reflects points through the line $x_2 = -x_1$ and then expands vertically by a factor of 2.

Let $[T] \in R^{2\times 2}$ be the matrix of T. $[T] = [T(e_1), T(e_2)]$

$$e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \to \begin{bmatrix} 0 \\ -1 \end{bmatrix} \to \begin{bmatrix} 0 \\ -2 \end{bmatrix} \quad \Rightarrow [T] = \begin{bmatrix} 0 & -1 \\ -2 & 0 \end{bmatrix}$$

$$e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \to \begin{bmatrix} -1 \\ 0 \end{bmatrix} \to \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

3. The linear transformation $T : R^2 \to R^3$ is such that $T(e_1) = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$, $T(e_2) = \begin{bmatrix} -2 \\ -4 \\ 1 \end{bmatrix}$, where e_1 and e_2 are the standard basis vectors in R^2. Is mapping T onto, one-to-one, or a bijection? Explain.

$$[T] = [T(e_1), T(e_2)] = \begin{bmatrix} 1 & -2 \\ 2 & -4 \\ -3 & 1 \end{bmatrix}.$$

$[T]$ is 3×2, hence <u>T is not onto since</u> # rows > # columns.

Since T is not onto, <u>T is not a bijection</u>.

$$\begin{bmatrix} 1 & -2 \\ 2 & -4 \\ -3 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & -2 \\ 0 & 0 \\ 0 & -5 \end{bmatrix} \to \begin{bmatrix} 1 & 2/5 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

All columns are pivot. <u>T is one-to-one</u>.