September 12, 2006 **Quiz 2** **Name:** Solution Key

1. Determine if vector u is a linear combination of vectors in $\{v_1,\ v_2,\ v_3\}$:

$$u = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix},\ v_1 = \begin{pmatrix} 0 \\ 2 \\ 7 \end{pmatrix},\ v_2 = \begin{pmatrix} 4 \\ 2 \\ 5 \end{pmatrix},\ v_3 = \begin{pmatrix} -12 \\ 2 \\ 13 \end{pmatrix}.$$

$$\begin{bmatrix} 0 & 4 & -12 & 0 \\ 2 & 2 & 2 & -1 \\ 7 & 5 & 13 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 1 & -1/2 \\ 0 & 1 & -3 & 0 \\ 7 & 5 & 13 & 0 \end{bmatrix} \begin{matrix} R_2 \cdot \frac{1}{2} \\ R_1 \cdot 1/4 \\ R_3 \end{matrix} \to \begin{bmatrix} 1 & 1 & 1 & -\frac{1}{2} \\ 0 & 1 & -3 & 0 \\ 0 & -2 & 5 & 7/2 \end{bmatrix} R_3 - 7R_1$$

$$\to \begin{bmatrix} 1 & 1 & 1 & -\frac{1}{2} \\ 0 & 1 & -3 & 0 \\ 0 & 0 & -1 & 7/2 \end{bmatrix} 2R_2 + R_3.$$

Yes, u is a linear combination of vectors in $\{v_1, v_2, v_3\}$

2. Determine if the vectors in $\{v_1, v_2, v_3, v_4\}$ generate R^3:

$$v_1 = \begin{pmatrix} 2 \\ -1 \\ 6 \end{pmatrix},\ v_2 = \begin{pmatrix} 7 \\ -4 \\ 23 \end{pmatrix},\ v_3 = \begin{pmatrix} -3 \\ 2 \\ 5 \end{pmatrix},\ v_4 = \begin{pmatrix} 12 \\ 1 \\ 4 \end{pmatrix}.$$

$$\begin{bmatrix} 2 & 7 & -3 & 12 \\ -1 & -4 & 2 & 1 \\ 6 & 23 & 5 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & 4 & -2 & -1 \\ 2 & 7 & -3 & 12 \\ 6 & 23 & 5 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & 4 & -2 & -1 \\ 0 & -1 & 1 & 14 \\ 0 & -1 & 17 & 10 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & 4 & -2 & -1 \\ 0 & 1 & -1 & -14 \\ 0 & 0 & 16 & -4 \end{bmatrix}$$

There is a pivot position in every row. The set generates R^3

3. Find the vectors that span the solution set of the homogeneous linear system

$$\begin{aligned} 3x_1 - 2x_2 - x_3 + 4x_4 &= 0 \\ -x_1 + x_2 + x_3 - 2x_4 &= 0 \end{aligned}$$

$$\begin{bmatrix} 3 & -2 & -1 & 4 \\ -1 & 1 & 1 & -2 \end{bmatrix} \to \begin{bmatrix} 1 & -1 & -1 & 2 \\ 3 & -2 & -1 & 4 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & -1 & -1 & 2 \\ 0 & 1 & 2 & -2 \end{bmatrix} \to \begin{array}{c} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & -2 \end{bmatrix} \\ x_1 \quad x_2 \quad x_3 \quad x_4 \end{array}$$

x_1, x_2 - basis

x_3, x_4 - free

$$\begin{aligned} x_4 &= s \\ x_3 &= t \\ x_2 &= -2t + 2s \\ x_1 &= -t \end{aligned}$$

$$x = t\begin{pmatrix} -1 \\ -2 \\ 1 \\ 0 \end{pmatrix} + s\begin{pmatrix} 0 \\ 2 \\ 0 \\ 1 \end{pmatrix}. \qquad \left\{ \begin{pmatrix} -1 \\ -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 0 \\ 1 \end{pmatrix} \right\}.$$