September 12, 2006 **Quiz #2** **Name:** Solution Key

1. Determine if vector u is a linear combination of vectors in $\{v_1, v_2, v_3\}$:

$$u = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}, \quad v_1 = \begin{pmatrix} 2 \\ 0 \\ 7 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 2 \\ 4 \\ 5 \end{pmatrix}, \quad v_3 = \begin{pmatrix} 2 \\ -12 \\ 13 \end{pmatrix}.$$

$$\begin{bmatrix} 2 & 2 & 2 & -1 \\ 0 & 4 & -12 & 0 \\ 7 & 5 & 13 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 1 & -1/2 \\ 0 & 1 & -3 & 0 \\ 7 & 5 & 13 & 0 \end{bmatrix} \begin{matrix} \tfrac{1}{2}R_1 \\ \tfrac{1}{4}R_2 \\ R_3 \end{matrix} \to \begin{bmatrix} 1 & 1 & 1 & -1/2 \\ 0 & 1 & -3 & 0 \\ 0 & -2 & 6 & 7/2 \end{bmatrix} R_3 - 7R_1$$

$$\to \begin{bmatrix} 1 & 1 & 1 & 1/2 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 7/2 \end{bmatrix} R_3 + 2R_2$$

The last row gives $0 = 7/2$, FALSE.
System is inconsistent $\Rightarrow$
u is not a linear combination of vectors in $\{v_1, v_2, v_3\}$.

2. Determine if the vectors in $\{v_1, v_2, v_3, v_4\}$ generate R^3:

$$v_1 = \begin{pmatrix} -1 \\ 2 \\ 6 \end{pmatrix}, \quad v_2 = \begin{pmatrix} -4 \\ 7 \\ 23 \end{pmatrix}, \quad v_3 = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix}, \quad v_4 = \begin{pmatrix} 1 \\ 12 \\ 4 \end{pmatrix}.$$

$$\begin{bmatrix} -1 & -4 & 2 & 1 \\ 2 & 7 & -3 & 12 \\ 6 & 23 & 5 & 4 \end{bmatrix} \to \begin{bmatrix} -1 & -4 & 2 & 1 \\ 0 & -1 & 1 & 14 \\ 0 & -1 & 17 & 10 \end{bmatrix} \to \begin{bmatrix} -1 & -4 & 2 & 1 \\ 0 & -1 & 1 & 14 \\ 0 & 0 & 16 & -4 \end{bmatrix}$$

There is a pivot position in every row.
The vectors generate $\mathbb{R}^3$.

3. Find the vectors that span the solution set of the homogeneous linear system

$$\begin{aligned} -x_1 + x_2 + x_3 - 2x_4 &= 0 \\ 3x_1 - 2x_2 - x_3 + 4x_4 &= 0 \end{aligned}$$

$$\begin{bmatrix} -1 & 1 & 1 & -2 \\ 3 & -2 & -1 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & -1 & -1 & 2 \\ 0 & 1 & 2 & -2 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & -2 \end{bmatrix}$$

x_1, x_2 - basic,
x_3, x_4 - free

$x_4 = s$
$x_3 = t$
$x_2 = -2t + 2s$
$x_1 = -t$

$$x = \begin{pmatrix} -1 \\ -2 \\ 1 \\ 0 \end{pmatrix} t + \begin{pmatrix} 0 \\ 2 \\ 0 \\ 1 \end{pmatrix} s. \quad \left\{ \begin{pmatrix} 1 \\ 2 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 0 \\ 1 \end{pmatrix} \right\}.$$