August 31, 2006 **Quiz 1** Name: Solution Key

1. Determine whether the following linear system is consistent or inconsistent, and if it is consistent then specify whether it has a unique solution or infinitely many solutions.

(a)
$$\begin{aligned} x - 2y + 3z &= 9 \\ 2x - 5y + 5z &= 17 \\ 3x - 7y + 8z &= 25 \end{aligned}$$

$$[A|b] = \begin{bmatrix} 1 & -2 & 3 & 9 \\ 2 & -5 & 5 & 17 \\ 3 & -7 & 8 & 25 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 3 & 9 \\ 0 & -1 & -1 & -1 \\ 0 & -1 & -1 & -2 \end{bmatrix} \begin{array}{l} \\ R_2 - 2R_1 \\ R_3 - 3R_1 \end{array}$$

$$\to \begin{bmatrix} 1 & -2 & 3 & 9 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The system is inconsistent since REF of the augmented matrix has the row $[0\ 0\ 0\ \underset{\neq 0}{1}]$.

(b)
$$\begin{aligned} 2x + 4y - 2z &= 0 \\ 3x + 5y &= 1 \end{aligned}$$

$$[A|b] = \begin{bmatrix} 2 & 4 & -2 & 0 \\ 3 & 5 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & -1 & 0 \\ 3 & 5 & 0 & 1 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & -1 & 3 & 1 \end{bmatrix}.$$

The system is consistent.
z is a free variable.
The system has ∞-many solutions.

2. Find the general solution of the linear system and write it in the vector-parametric form.

$$\begin{aligned} 2x + 5y - z + 2w &= -6 \\ x + 5y + 2z + 6w &= -3 \end{aligned}$$

$$[A|b] = \begin{bmatrix} 2 & 5 & -1 & 2 & -6 \\ 1 & 5 & 2 & 6 & -3 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & 5 & 2 & 6 & -3 \\ 0 & -5 & -5 & -10 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 5 & 2 & 6 & -3 \\ 0 & 1 & 1 & 2 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -3 & -4 & -3 \\ 0 & 1 & 1 & 2 & 0 \end{bmatrix}$$

x, y - basic variables, z, w - free variables.

$$\begin{aligned} x &= -3 + 3z + 4w \\ y &= \quad - z - 2w \\ z &= z \\ w &= w. \end{aligned} \Rightarrow \boxed{\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \\ 0 \\ 0 \end{pmatrix} + z\begin{pmatrix} 3 \\ -1 \\ 1 \\ 0 \end{pmatrix} + w\begin{pmatrix} 4 \\ -2 \\ 0 \\ 1 \end{pmatrix} \quad z, w \in \mathbb{R}.}$$