August 31, 2006 **Quiz 1** **Name:** Solution Key

1. Determine whether the following linear system is consistent or inconsistent, and if it is consistent then specify whether it has a unique solution or infinitely many solutions.

(a)
$$\begin{aligned} 2x - 5y + 5z &= 17 \\ x - 2y + 3z &= 9 \\ x - 3y + 2z &= 10 \end{aligned}$$

$$[A|b] = \begin{bmatrix} 2 & -5 & 5 & 17 \\ 1 & -2 & 3 & 9 \\ 1 & -3 & 2 & 10 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 3 & 9 \\ 2 & -5 & 5 & 17 \\ 1 & -3 & 2 & 10 \end{bmatrix} \begin{array}{l} R_2 \\ R_1 \\ R_3 \end{array}$$

$$\to \begin{bmatrix} 1 & -2 & 3 & 9 \\ 0 & -1 & -1 & -1 \\ 0 & -1 & -1 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 3 & 9 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The linear system is inconsistent since the REF of A has the row $[0, 0, 0, 1]$.

(b)
$$\begin{aligned} 5x + 9y - 2z &= 2 \\ 3x + 6y &= -3 \end{aligned}$$

$$[A|b] = \begin{bmatrix} 5 & 9 & -2 & 2 \\ 3 & 6 & 0 & -3 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & 9/5 & -2/5 & 2/5 \\ 3 & 6 & 0 & -3 \end{bmatrix} \to \begin{bmatrix} 1 & 9/5 & -2/5 & 2/5 \\ 0 & 3/5 & * & * \end{bmatrix}$$

The linear system is consistent since there is no row $[0\ 0\ 0\ *]$. x_3 is a free variable. Therefor, there are ∞-many solutions.

2. Find the general solution of the linear system and write it in the vector-parametric form.

$$\begin{aligned} 3x + 4y + w &= 3 \\ 2x + y - z + 2w &= 1 \\ x + 5y + 2z + 6w &= 2 \end{aligned}$$

$$[A|b] = \begin{bmatrix} 3 & 4 & 1 & 0 & 3 \\ 2 & 1 & -1 & 2 & 1 \\ 1 & 5 & 2 & 6 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 5 & 2 & 6 & 2 \\ 2 & 1 & -1 & 2 & 1 \\ 3 & 4 & 1 & 0 & 3 \end{bmatrix} \begin{array}{l} R_3 \\ \\ R_1 \end{array}$$

$$\to \begin{bmatrix} 1 & 5 & 2 & 6 & 2 \\ 0 & -9 & -5 & -10 & -3 \\ 0 & -11 & -5 & -18 & -3 \end{bmatrix} \begin{array}{l} \\ R_2 \leftarrow R_2 - 2R_1 \\ R_3 \leftarrow R_3 - 3R_1 \end{array} \to \begin{bmatrix} 1 & 5 & 2 & 6 & 2 \\ 0 & -9 & -5 & -10 & -3 \\ 0 & -2 & 0 & -8 & 0 \end{bmatrix} R_3 - R_2$$

$$\to \begin{bmatrix} 1 & 5 & 2 & 6 & 2 \\ 0 & 1 & 0 & 4 & 0 \\ 0 & -9 & -5 & -10 & -3 \end{bmatrix} R_3 \cdot \left(-\tfrac{1}{2}\right) \to \begin{bmatrix} 1 & 5 & 2 & 6 & 2 \\ 0 & 1 & 0 & 4 & 0 \\ 0 & 0 & -5 & 26 & -3 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & 5 & 2 & 6 & 2 \\ 0 & 1 & 0 & 4 & 0 \\ 0 & 0 & 1 & -\frac{26}{5} & \frac{3}{5} \end{bmatrix} \to \begin{bmatrix} 1 & 5 & 0 & -22/5 & 4/5 \\ 0 & 1 & 0 & 4 & 0 \\ 0 & 0 & 1 & -\frac{26}{5} & \frac{3}{5} \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & -\frac{122}{5} & 4/5 \\ 0 & 1 & 0 & 4 & 0 \\ 0 & 0 & 1 & -\frac{26}{5} & \frac{3}{5} \end{bmatrix}$$

x_1, x_2, x_3 – basic, x_4 – free

$x_1 = \frac{4}{5} + \frac{122}{5}s$ $\quad x_3 = \frac{3}{5} + \frac{26}{5}s$, $\quad x = \begin{pmatrix} 4/5 \\ 0 \\ 3/5 \\ 0 \end{pmatrix} + s \begin{pmatrix} 122/5 \\ -4 \\ 26/5 \\ 1 \end{pmatrix}$, $s \in \mathbb{R}$.

$x_2 = -4s$ $\quad x_4 = s$.