

Problem 4. Solve the following recurrence relations.

1. $a_n = 5a_{n-1} - 4a_{n-2}, n \geq 2, a_0 = 0, a_1 = 1.$

Characteristic equation is $r^2 - 5r + 4 = 0$

$$r_{1,2} = \frac{5 \pm \sqrt{25 - 16}}{2} = \frac{5 \pm 3}{2} \Rightarrow r_1 = 1, r_2 = 4$$

$$a_n = \alpha_1 1^n + \alpha_2 4^n = \alpha_1 + \alpha_2 4^n.$$

$$\left. \begin{array}{l} 0 = a_0 = \alpha_1 + \alpha_2 \\ 1 = a_1 = \alpha_1 + 4\alpha_2 \end{array} \right\} \Rightarrow 3\alpha_2 = 1 \Rightarrow \begin{cases} \alpha_2 = \frac{1}{3} \\ \alpha_1 = -\frac{1}{3} \end{cases}$$

$$a_n = -\frac{1}{3} + \frac{1}{3} 4^n = \underline{\underline{\frac{1}{3}(4^n - 1)}}, n \geq 0.$$

2. $a_n + 6a_{n-1} + 9a_{n-2} = 0, n \geq 2, a_0 = 3, a_1 = -3.$

$$r^2 + 6r + 9 = 0 \quad r_{1,2} = \frac{-6 \pm \sqrt{36 - 36}}{2} = -3$$

$$a_n = (\alpha_1 + n\alpha_2)(-3)^n.$$

$$3 = a_0 = \alpha_1$$

$$-3 = a_1 = (3 + \alpha_2)(-3) \Rightarrow 3 + \alpha_2 = 1 \Rightarrow \alpha_2 = -2.$$

$$a_n = (3 - 2n)(-3)^n, n \geq 0.$$