Problem 3. Use the binomial theorem for the following.

1. Expand $(2a-b)^5$.

```
      1
    1   1
  1   2   1
1   3   3   1
1 4   6   4 1
1 5 10  10  5 1
```

$$(2a-b)^5 = (2a)^5 + 5(2a)^4(-b) + 10(2a)^3(-b)^2 + 10(2a)^2(-b)^3 + 5(2a)(-b)^4 + (-b)^5$$
$$= 32a^5 - 80a^4b + 80a^3b^2 - 40a^2b^3 + 10ab^4 - b^5.$$

2. Find the coefficient of x^2y^4 in the binomial expansion of $(2x-y)^6$.

$$C(6,2)(2x)^2(-y)^4 = \frac{6!}{2!4!} \cdot 4x^2y^4 = 60x^2y^4$$

$$\boxed{60}$$

3. Prove

$$C(5,0) + C(5,1) + C(5,2) + C(5,3) + C(5,4) + C(5,5) = 2^5.$$

$$2^5 = (1+1)^5 = \sum_{j=0}^{5} C(5,j)\, 1^j\, 1^{5-j} = \sum_{j=0}^{5} C(5,j).$$