

## Test 3

Problem	1	2	3	4	Score
Points	/25	/10	/15	/20	/70

NAME: Solution Key

Show all your work for full credit.

## Problem 1.

1. How many 12-bit binary strings have exactly four ones?

# 4-combinations out of a set with 12 elements

$$C(12, 4) = \frac{12!}{4! 8!} = \frac{9 \cdot \overset{5}{\cancel{10}} \cdot 11 \cdot 12}{1 \cdot 2 \cdot \cancel{3} \cdot 4} = 9 \cdot 5 \cdot 11 = \underline{\underline{495}}$$

2. How many 12-bit binary strings have exactly four ones, and none of these ones are adjacent to each other?

There must be 3 zeros between ones

$$12 - 3 = 9$$

# 4-combinations out of a set with 9 elements

$$C(9, 4) = \frac{9!}{4! 5!} = 7 \cdot 2 \cdot 9 = \underline{\underline{126}}$$

3. How many permutations of the seven letters A, B, C, D, E, F, and G have E in one of its first two positions?

# 6 permutations (E in the 1st position)

plus # 6 permutations (E in the 2nd position)

$$P(6, 6) = \frac{6!}{(6-6)!} = 6! \Rightarrow 2 \cdot 6! = 2 \cdot 720 = \underline{\underline{1,440}}$$

4. Assume that you have 30 different books. In how many ways you can give 4 books to a friend.

# 4-combinations out of a set of 30 elements

$$C(30, 4) = \frac{30!}{4! 26!} = 27 \cdot 7 \cdot 29 \cdot 5 = \underline{\underline{27,405}}$$

5. How many 5 letter words with different letters that begin with D and end with E can be formed from the set {A, B, C, D, E, F, G, H, I}.

2 letters are fixed in a 5-letter word  $\rightarrow 5 - 2 = 3$ 

There are 9 letters in the set of letters.

 $9 - 2 = 7$ . We need # 3-permutation of a set with 7 elements

$$P(7, 3) = \frac{7!}{(7-3)!} = 5 \cdot 6 \cdot 7 = \underline{\underline{210}}$$