Problem 5. Prove

$$1 - 2 + 2^2 - 2^3 + \cdots + (-1)^n 2^n = \frac{(-1)^n 2^{n+1} + 1}{3} \quad , n = 0, 1, 2, \ldots$$

$n = 0 \Rightarrow \dfrac{(-1)^0 2^1 + 1}{3} = 1 \Rightarrow$ True.

Assume

$$1 - 2 + \ldots + (-1)^k 2^k = \frac{(-1)^k 2^{k+1} + 1}{3}.$$

Consider

$$1 - 2 + \ldots + (-1)^k 2^k + (-1)^{k+1} 2^{k+1} = \frac{(-1)^k 2^{k+1} + 1}{3} + (-1)^{k+1} 2^{k+1}$$

$$= \frac{-(-1)^{k+1} 2^{k+1} + 1 + 3(-1)^{k+1} 2^{k+1}}{3} = \frac{2(-1)^{k+1} 2^{k+1} + 1}{3} = \frac{(-1)^{k+1} 2^{k+2} + 1}{3}$$

True

Problem 6 Give a recursive definition of the function $f(n) = 5n + 2$, $n \in Z^+$. $n \geq 1$

$$f(n) = 5n + 2 = 5(n-1) + 7 = 5(n-1) + 2 + 5 = f(n-1) + 5.$$

<u>Basis step</u>: $f(1) = 7$

<u>Inductive step</u>: $f(n) = f(n-1) + 5, \ n \geq 2.$