Problem 4. Find:

1. 81 mod 17

$81 = 17 \cdot 4 + 13 \Rightarrow 81 \bmod 17 = 13$

2. -78 mod 12

$-78 = 12(-7) + 6 \Rightarrow -78 \bmod 12 = 6$

3. $\gcd(4^3 7^{13} 8^8, 2^3 4^4 7^5)$ Let $a = 4^3 7^{13} 8^8$ and $b = 2^3 4^4 7^5$.

$a = (2^2)^3 7^{13} (2^3)^8 = 2^{6+24} 7^{13} = 2^{30} 7^{13}$

$b = 2^3 2^8 \cdot 7^5 = 2^{11} 7^5$

$\Rightarrow \gcd(a, b) = 2^{\min(30, 11)} 7^{\min(13, 5)} = 2^{11} 7^5$

4. $\operatorname{lcm}(4^3 7^{13} 8^8, 2^3 4^4 7^5)$

$\operatorname{lcm}(a, b) = 2^{\max(30, 11)} 7^{\max(13, 5)} = 2^{30} 7^{13}$

5. The prime factorization of 375. $375 \div 3 = 125 = 5^3$

$375 = 3 \cdot 5^3$

6. Using the Eucledian algorithm, gcd(72, 390)

$\gcd(390, 72) = \gcd(72, 30) = \gcd(30, 12) = \gcd(12, 6) = 6$