Problem 7. Determine if the function $f : R \to R$, $f(x) = 2x^3 + 1$, is a bijection.

f is one-to-one.

$f(x)$ is strictly increasing for any $x \neq 0$, since $f'(x) = 6x^2 > 0$.

Or, suppose $f(x_1) = f(x_2)$.

$2x_1^3 + 1 = 2x_2^3 + 1$

$x_1^3 = x_2^3 \Rightarrow x_1 = x_2 \Rightarrow f$ is one-to-one.

f is onto

Take any $y \in R$. Consider $2x^3 + 1 = y$

$\Rightarrow x^3 = \frac{y-1}{2} \Rightarrow x = \sqrt[3]{\frac{y-1}{2}} \Rightarrow f$ is onto.

Problem 8. Let $a \neq 0$. Rewrite the sum $\sum_{i=-3}^{5}(3i+1)a^{i+2}$ as the sum with the lower limit 0.

Let $j = i + 3$. Then $j = 0$ if $i = -3$.

Using $i = j - 3$, we get

$$\sum_{i=-3}^{5}(3i+1)a^{i+2} = \sum_{j=0}^{8}[3(j-3)+1]a^{j-3+2} = \sum_{j=0}^{8}[3j-8]a^{j-1}.$$