Problem 5 Prove the proposition "*A positive integer n is even if and only if $3n^2 + 8$ is even.*"

Proof. $\rightarrow$ "If $n > 0$ is even, then $3n^2+8$ is even".

Let $n = 2k$. for $k \in \mathbb{Z}$.

$3n^2+8 = 3(2k)^2+8 = 2(6k^2+4)$.

Since $6k^2+4 \in \mathbb{Z}$, $3n^2+8$ is even. ∎

$\leftarrow$ "If $3n^2+8$ is even, then n is even"

We prove the counterpositive: If n is odd, then $3n^2+8$ is odd.

Let $n = 2k+1$

$3n^2+8 = 3(2k+1)^2+8 = 3(4k^2+4k+1)+8 = 2(6k^2+6k+5)+1$.

Since $6k^2+6k+5 \in \mathbb{Z}$, $3n^2+8$ is odd. ∎

Problem 6. Prove the set relation $(A \cap B) \cup (A \cap \overline{B}) = A$.

$(A \cap B) \cup (A \cap \overline{B}) = A \cap (B \cup \overline{B}) = A \cap U = A$

Here U is a universal set and we used distributive law.