

Test 1

Problem	1	2	3	4	5	6	7	8	Grade
Points	/10	/10	/10	/10	/10	/10	/10	/10	/80

NAME:

Solution Key

Show all your work for full credit.

Problem 1. Determine if the proposition $(p \rightarrow q) \wedge (\neg p \rightarrow q) \leftrightarrow q$ is a tautology.

p	q	$\neg p$	$p \rightarrow q$	$\neg p \rightarrow q$	$(p \rightarrow q) \wedge (\neg p \rightarrow q)$	$(p \rightarrow q) \wedge (\neg p \rightarrow q) \leftrightarrow q$
T	T	F	T	T	T	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	F	T	T	F	F	T

The proposition is a tautology.

Problem 2. Let $P(x, y)$ be the predicate " x and y are reals such that $x + 2y = 5$." Determine the truth values of the statements $\exists x \forall y P(x, y)$ and $\forall x \exists y P(x, y)$.

① Consider $\exists x \forall y P(x, y)$

let $y=0$. Then $x=5$

let $y=1$. Then $x=3$

Since $5 \neq 3$, $\nexists x$ such that $P(x, 0) \equiv P(x, 1) \equiv T \Rightarrow \exists x \forall y P(x, y) \equiv F$

② For any x , let $y = \frac{5-x}{2}$.

Then $(x + 2y = 5) \equiv (x + 2 \cdot \frac{5-x}{2} = 5) \equiv (5 = 5) \equiv T$.

$\Rightarrow \forall x \exists y P(x, y) \equiv T$.