

Quiz 3

15 pt.

Name:

Solution Key

1. Prove $A - (B \cup C) = (A - B) \cap (A - C)$.

$$\begin{aligned} \{x \in A - (B \cup C)\} &\leftrightarrow \{x \in A \text{ and } x \notin B \cup C\} \leftrightarrow \{x \in A \text{ and } x \in \overline{B \cup C}\} \\ &\leftrightarrow \{x \in A \text{ and } x \in \overline{B} \cap \overline{C}\} \leftrightarrow \{x \in A \text{ and } x \in \overline{B} \text{ and } x \in \overline{C}\} \\ &\leftrightarrow \{(x \in A \text{ and } x \in \overline{B}) \text{ and } (x \in A \text{ and } x \in \overline{C})\} \\ &\leftrightarrow \{(x \in A \text{ and } x \notin B) \text{ and } (x \in A \text{ and } x \notin C)\} \\ &\leftrightarrow \{x \in A - B \text{ and } x \in A - C\} \leftrightarrow \{x \in (A - B) \cap (A - C)\} \end{aligned}$$

2. Find the power set of the set $A = \{1, 2, 3\}$.

$$P(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}\}$$

3. Find the cardinality of the set $\{\emptyset, \{\emptyset, a\}, \{\emptyset, a, \{\emptyset, a\}\}\}$.

The set consists of 3 distinct elements.

The cardinality of the set is 3.

4. Is the function $f: \mathbb{N} \rightarrow \mathbb{N}$ defined by $f(x) = 2x - 1$, one-to-one? Explain. (Recall that \mathbb{N} is the set of nonnegative integers).

$$\mathbb{N} = \{0, 1, 2, \dots\}$$

$$f(0) = 2 \cdot 0 - 1 = -1 \notin \mathbb{N}$$

f is not a function from \mathbb{N} to \mathbb{N} .

f is not one-to-one function.

Suppose $f: \mathbb{N} \rightarrow \mathbb{Z}$.

For any $x_1 \neq x_2$ in \mathbb{N} ,

$$f(x_1) - f(x_2) = 2x_1 - 1 - (2x_2 - 1) = 2(x_1 - x_2) \neq 0$$

$\Rightarrow f(x_1) \neq f(x_2)$ and

$f: \mathbb{N} \rightarrow \mathbb{Z}$ is one-to-one.