Quiz 2 **Name:** Solution key

1. Assuming that x does not occur in proposition A as a free variable, establish the logical equivalence

$$\forall x(P(x) \to A) \equiv \exists x P(x) \to A.$$

(Hint: Consider the two cases $A \equiv F$ and $A \equiv T$, and use the relation $p \to F \equiv \neg p$). (*)

Let $A \equiv F$. Prove $\forall x(P(x) \to F) \equiv (\exists x(P(x))) \to F$

Using (*),

$\forall x(\neg P(x)) \equiv \neg(\exists x(P(x)))$

$\forall x(\neg P(x)) \equiv \forall x(\neg P(x))$ - Tautology.

Let $A \equiv T$. Prove $\forall x(P(x) \to T) \equiv (\exists x(P(x))) \to T$

Note, $(p \to T) \equiv T$.

Therefore,

$\forall x(T) \equiv T$

$T \equiv T$ - tautology

2. Consider the folowing argument: "All foods that are healthy to eat do not taste good," (1) "Tofu is healthy to eat," (2) "I eat food only if it tastes good," (3) with the conclusion "I do not eat tofu." Build an argument form and prove its validity using the rules of inference.

Let: $h(x) \equiv$ "x is healthy to eat",

$e(x) \equiv$ "I eat x",

$t(x) \equiv$ "x tastes good".

Domain: all foods.

Argument form:

(1) $\forall x(h(x) \to \neg t(x))$
(2) $h(\text{tofu})$
(3) $\forall x(e(x) \to t(x))$

$\}$ Hypotheses

$\therefore \neg e(\text{tofu})$ $\}$ conclusion

Proof of validity:

$\forall x(h(x) \to \neg t(x))$	Hypothesis
$x \equiv \text{tofu}$	Hypothesis
$h(\text{tofu}) \to \neg t(\text{tofu})$	Universal instantiation
$h(\text{tofu})$	Hypothesis
$\neg t(\text{tofu})$	Modus ponens
$\forall x(e(x) \to t(x))$	Hypothesis
$\neg e(\text{tofu})$	Universal Modus tollens. (conclusion).