Problem 6. Test the integral for convergence:

$$\int_1^\infty \frac{e^{-t}}{t}\,dt$$

Use Direct Comparison Test with $g(t) = e^{-t}$.

$$\frac{e^{-t}}{t} \le e^{-t} \text{ for } t \ge 1$$

$$\int_1^\infty e^{-t}\,dt = \lim_{A\to\infty}\left(-e^{-A} + e^{-1}\right) = \frac{1}{e} \Rightarrow \text{convergent}$$

$$\Rightarrow \int_1^\infty \frac{e^{-t}}{t}\,dt \text{ is convergent.}$$

Problem 7. Evaluate the integral if it is convergent:

$$\int_0^\infty xe^{-x}\,dx = \lim_{A\to\infty}\int_0^A xe^{-x}\,dx = \lim_{A\to\infty}\int_0^A x\left(-e^{-x}\right)'\,dx$$

$$= \lim_{A\to\infty}\left(-xe^{-x}\Big|_0^A\right) + \lim_{A\to\infty}\int_0^A e^{-x}\,dx$$

$$= \lim_{A\to\infty}\left(-\frac{A}{e^A}\right) - \lim_{A\to\infty} e^{-x}\Big|_0^A$$

$$= -\lim_{A\to\infty}\frac{1}{e^A} + e^{-0} = 1. \text{ Convergent.}$$