

Problem 4. Evaluate the integral

$$\int \frac{4x}{x^2+4} dx = \int \frac{4 dx}{x^2+4} = \int \frac{dx}{\left(\frac{x}{2}\right)^2+1}$$

$$= \left| \begin{array}{l} x = 2u \\ dx = 2 du \end{array} \right| = 2 \int \frac{du}{u^2+1} = 2 \tan^{-1} u + C = 2 \tan^{-1} \frac{x}{2} + C$$

Problem 5. Explain why the following integral is improper and evaluate it if convergent:

$$\int_1^2 \frac{dx}{(x-1)^{3/5}} \quad f(x) = \frac{1}{(x-1)^{3/5}} \rightarrow +\infty \text{ as } x \rightarrow 1^+$$

Integrand is unbounded \Rightarrow integral is improper.

$$\int_1^2 \frac{dx}{(x-1)^{3/5}} = \left| \begin{array}{l} x-1=u \\ dx=du \end{array} \right| = \int_0^1 \frac{du}{u^{3/5}} = \lim_{\epsilon \rightarrow 0^+} \int_{\epsilon}^1 \frac{du}{u^{3/5}}$$

$$= \lim_{\epsilon \rightarrow 0^+} \frac{u^{1-3/5}}{1-3/5} \Big|_{\epsilon}^1 = \frac{5}{2} \left(1 - \lim_{\epsilon \rightarrow 0} \epsilon^{2/5} \right) = \frac{5}{2}$$