

Problem 2. Evaluate the integral

$$\int \tan^3 x \sec^3 x dx = \int \tan^2 x \sec^2 x (\tan x \sec x dx)$$

$$= \left. \begin{array}{l} u = \sec x \\ du = \tan x \sec x dx \\ \tan^2 x = \sec^2 x - 1 \end{array} \right\} = \int (u^2 - 1) u^2 du$$

$$= \int (u^4 - u^2) du = \frac{u^5}{5} - \frac{u^3}{3} + C = \frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C.$$

Problem 3. Evaluate the integral

$$\int \frac{dt}{\sqrt{1-4t^2}} dx = \left| \begin{array}{l} t = \frac{x}{2} \\ dt = \frac{dx}{2} \end{array} \right| = \frac{1}{2} \int \frac{dx}{\sqrt{1-x^2}}$$

$$= \left| \begin{array}{l} x = \sin \theta \\ dx = \cos \theta d\theta \end{array} \right| = \frac{1}{2} \int \frac{\cos \theta d\theta}{\cos \theta} = \frac{1}{2} \int d\theta$$

$$= \frac{1}{2} \theta + C = \frac{1}{2} \sin^{-1} x + C = \frac{1}{2} \sin^{-1}(2t) + C.$$