Problem 6. Solve the initial value problem

$$xy' = (x+1)(y-1)^2, \quad x > 1, \quad y(1) = 3.$$

$$x\frac{dy}{dx} = (x+1)(y-1)^2$$

$$\frac{dy}{(y-1)^2} = \frac{x+1}{x}\,dx \Rightarrow \int \frac{dy}{(y-1)^2} = \int\left(1+\frac{1}{x}\right)dx.$$

$$\int \frac{dy}{(y-1)^2} = \left|\begin{matrix} z = y-1 \\ dz = dy\end{matrix}\right| = \int z^{-2}\,dz = -\frac{1}{z} + C = -\frac{1}{y-1} + C = \frac{1}{1-y} + C.$$

$$\int\left(1+\frac{1}{x}\right)dx = x + \ln|x| + C. \quad (x>1)$$

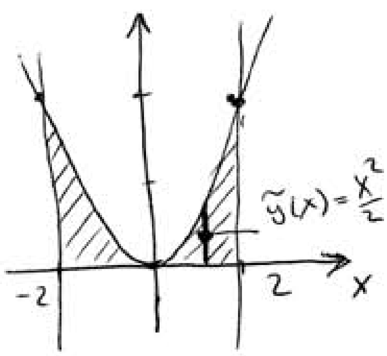
$$\frac{1}{1-y} = x + \ln x + C \Rightarrow y - 1 = -\frac{1}{x+\ln x + C} \Rightarrow y = 1 - \frac{1}{x+\ln x+C}$$

$$y(1) = 3 = 1 - \frac{1}{1+\ln 1 + C} \Rightarrow 3 = 1 - \frac{1}{1+C} \Rightarrow C+1 = -\frac{1}{2}$$

$$\Rightarrow C = -\frac{3}{2}. \quad \boxed{y(x) = 1 - \frac{1}{x+\ln x - \frac{1}{2}}}$$

Problem 7. Find the centroid of a thin flat plate covering the region enclosed by the x-axis, the lines $x = -2$ and $x = 2$, and the parabola $y = x^2$.

$$h(x) = x^2 - 0 = x^2$$



$$M = \int_{-2}^{2} h(x)\,dx = \int_{-2}^{2} x^2\,dx = \frac{x^3}{3}\Big|_{-2}^{2} = \frac{16}{3}.$$

$$M_y = \int_{-2}^{2} h(x)\,x\,dx = \int_{-2}^{2} x^3\,dx = \frac{x^4}{4}\Big|_{-2}^{2} = 0$$

$$M_x = \int_{-2}^{2} h(x)\,\tilde{y}(x)\,dx = \int_{-2}^{2} x^2\cdot\frac{x^2}{2}\,dx = \frac{1}{2}\,\frac{x^5}{5}\Big|_{-2}^{2} = \frac{32}{5}.$$

$$\bar{x} = \frac{M_y}{M} = 0, \quad \bar{y} = \frac{M_x}{M} = \frac{32/5}{16/3} = \frac{6}{5}. \quad \boxed{\left(0, \frac{6}{5}\right)}$$