

Problem 4. Find the area of the surface generated by revolving about the x -axis the parametric curve

$$x(t) = \cos t, \quad y(t) = 2 + \sin t, \quad 0 \leq t \leq 2\pi. \quad A = 2\pi \int_a^b y(t) \sqrt{(x'(t))^2 + (y'(t))^2} dt$$

$$x'(t) = -\sin t, \quad y'(t) = \cos t.$$

$$A = 2\pi \int_0^{2\pi} (2 + \sin t) \sqrt{\sin^2 t + \cos^2 t} dt = 2\pi \int_0^{2\pi} (2 + \sin t) dt$$

$$= 2\pi \left(4\pi - \cos t \Big|_0^{2\pi} \right) = 2\pi (4\pi - 0) = 8\pi^2.$$

Problem 5. A force of 200 N will stretch the garage door spring 0.8 m beyond its unstressed length. How far will a 300 N force stretch the spring. How much work does it take to stretch the spring this far from its unstressed length? Hint: Use Hook's law $F = kx$.

$$k = \frac{F}{x} \Rightarrow k = \frac{200}{0.8} = 250 \left(\frac{N}{m} \right) \Rightarrow F(x) = 250x.$$

$$x = \frac{F}{k} \Rightarrow x = \frac{300}{250} = \frac{6}{5} \text{ (m)}$$

$$W = \int_0^{\ell} F(x) dx = \int_0^{6/5} 250x dx = 250 \cdot \frac{x^2}{2} \Big|_0^{6/5} = 125 \cdot \left(\frac{6}{5} \right)^2$$

$$= \frac{125}{25} \cdot 36 = 36 \cdot 5 = 180 \text{ (J)}.$$