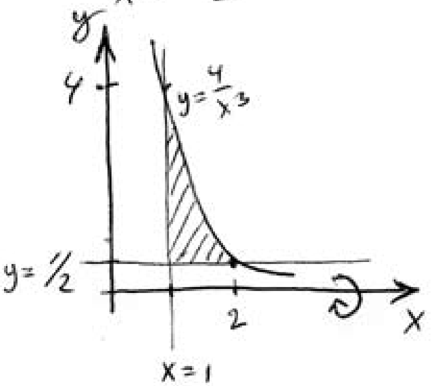
Problem 2. Find the volume of the solid generated by revolving the plane region bounded by th curve $y = 4/x^3$ and the lines $y = 1/2$ and $x = 1$ about the x-axis. Sketch the plane region.

Find the intersection point of $y = \frac{4}{x^3}$ and $y = \frac{1}{2}$.

$\frac{4}{x^3} = \frac{1}{2} \Rightarrow x^3 = 8 \Rightarrow x = 2.\ y(2) = \frac{1}{2}$



Use the <u>washers</u> method

$$V = \pi \int_1^2 \left[\left(\frac{4}{x^3}\right)^2 - \left(\frac{1}{2}\right)^2\right] dx = \pi \int_1^2 \left[\frac{16}{x^6} - \frac{1}{4}\right] dx$$

$$= \pi\left(-\frac{16}{5}x^{-5} - \frac{1}{4}x\right)\Big|_1^2$$

$$= \pi\left(-\frac{16}{5}\cdot\frac{1}{32} - \frac{1}{2} + \frac{16}{5} + \frac{1}{4}\right) = \pi\left(\frac{16}{5}\cdot\frac{31}{32} - \frac{1}{4}\right)$$

$$= \pi\,\frac{62-5}{20} = \frac{57}{20}\pi.$$

Problem 3. Find the length of the curve given by the parametric equations

$$x(t) = t^2/2, \quad y(t) = (2t+1)^{3/2}/3, \quad t \in [0, 4].$$

$x' = t,\ y' = \frac{1}{3}\cdot\frac{3}{2}\cdot 2(2t+1)^{1/2} = (2t+1)^{1/2}.$

$$L = \int_0^4 \sqrt{t^2 + ((2t+1)^{1/2})^2}\, dt = \int_0^4 \sqrt{t^2+2t+1}\, dt = \int_0^4 \sqrt{(t+1)^2}\, dt$$

$$= \int_0^4 (t+1)\, dt = \left(\frac{t^2}{2} + t\right)\Big|_0^4 = 8 + 4 = 12.$$