November 6, 2009 **Quiz 4** Name: Solution Key

1. Find the sum of the series:

$$\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2)}$$

$$, \quad \frac{1}{n^2} - \frac{1}{(n+1)^2} = \frac{(n+1)^2 - n^2}{n^2(n+1)^2} = \frac{2n+1}{n^2(n+1)^2}.$$

$$= \sum_{n=1}^{\infty} \left(\frac{1}{n^2} - \frac{1}{(n+1)^2} \right)$$

$$= \lim_{N\to\infty} \sum_{n=1}^{N} \left(\frac{1}{n^2} - \frac{1}{(n+1)^2} \right) = \lim_{N\to\infty} \left(1 - \frac{1}{(N+1)^2} \right) = 1$$

The sum is 1.

2. Does the serious converge? Explain.

$$\sum_{n=1}^{\infty} \frac{(\sin n)^2}{n^2}$$

Use comparison test.

$$a_n = \frac{(\sin n)^2}{n^2} \le \frac{1}{n^2} = b_n.$$

The series $\sum b_n = \sum \frac{1}{n^2}$ converges (p-series, $p = 2 > 1$).

$\Rightarrow \sum a_n$ converges.

3. Use the integral test to determine if the series converges:

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$$

$$f(x) = \frac{1}{x(\ln x)^2}$$

$$\int_2^{\infty} f(x)\,dx = \lim_{A\to\infty} \int_2^A f(x)\,dx = \lim_{A\to\infty} \int_2^A \frac{dx}{x(\ln x)^2} = \lim_{A\to\infty} \int_2^A \frac{d\ln x}{(\ln x)^2}$$

$$= -\lim_{A\to\infty} \frac{1}{\ln x}\Big|_2^A = -\lim_{A\to\infty} \frac{1}{\ln A} + \frac{1}{\ln 2} = \frac{1}{\ln 2}$$ converges.

The series converges.