October 5, 2009 **Quiz 3** Name: Solution Key

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1. Evaluate the integral $\int t^2 e^{4t}\, dt$.

$$I = \int t^2 e^{4t}\,dt = \frac{1}{4}\int t^2 (e^{4t})'\,dt = \frac{t^2}{4}e^{4t} - \frac{1}{2}\int t e^{4t}\,dt$$

$$\int t e^{4t}\,dt = \frac{1}{4}\int t(e^{4t})'\,dt = \frac{t}{4}e^{4t} - \frac{1}{4}\int e^{4t}\,dt = \frac{t}{4}e^{4t} - \frac{1}{16}e^{4t} + C$$

$$I = \frac{t^2}{4}e^{4t} - \frac{1}{8}\left(t e^{4t} - \frac{1}{4}e^{4t}\right) + C$$

$$= \frac{e^{4t}}{8}\left[2t^2 - t + \frac{1}{4}\right] + C.$$

2. Evaluate the integral. (Hint: $\sin\frac{\pi}{6} = \frac{1}{2}$, $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$.)
$\int_0^{1/\sqrt{2}} 2x\ \sin^{-1}(x^2)\,dx.$

$$= \left| \begin{matrix} u = x^2 \\ du = 2x\,dx \end{matrix} \right| = \int_0^{1/2} \sin^{-1} u\,du = \left| \begin{matrix} z = \sin^{-1} u, \ \sin z = u \\ du = \cos z\,dz \end{matrix} \right|$$

$$= \int_0^{\pi/6} z\cos z\,dz = \int_0^{\pi/6} z\,d\sin z = z\sin z\Big|_0^{\pi/6} + \cos z\Big|_0^{\pi/6}$$

$$\cos z\Big|_0^{\pi/6} = -\int_0^{\pi/6}\sin z\,dz$$

$$= \frac{\pi}{6}\cdot\frac{1}{2} + \frac{\sqrt{3}}{2} - 1 = \cancel{\frac{1}{2}\left(\frac{\pi}{6} - 2\right)} = \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$$

3. Evaluate the integral $\int_0^{\pi/2} \sin 2x\ \cos 3x\,dx$.

$$\sin 2x\cos 3x = \frac{1}{2}\left[\sin(-x) + \sin 5x\right]$$

$$I = \frac{1}{2}\int_0^{\pi/2}\left[\sin 5x - \sin x\right]dx = \frac{1}{2}\left(-\frac{1}{5}\cos 5x + \cos x\right)\Big|_0^{\pi/2}$$

$$= \frac{1}{2}\left(\frac{1}{5} - 1\right) = -\frac{2}{5}.$$